

CRITICS 2017 SPRING DOCTORAL SCHOOL

18-22 APRIL 2017, LECCE, ITALY.

Preliminary Program



CRITICS
Critical Transitions in Complex Systems



**UNIVERSITÀ
DEL SALENTO**


Dipartimento di Matematica e Fisica
"Ennio De Giorgi"

Tuesday 18 April

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|---------------|--------------------------|---|
| 09:00 - 9:30 | Welcome and registration | |
| 09:30 - 11:30 | Gianfausto Salvadori | Lecture 1: Multivariate modeling via Copulas: theory and practice |
| 11:30 - 12:00 | | Coffee Break |
| 12:00 - 13:00 | Martin Rasmussen | Spectral Theory for Nonautonomous Differential Equations |
| 13:00 - 15:00 | | Lunch |
| 15:00 - 15:30 | | Transport to the Dept. of Mathematics |
| 15:30 - 17:30 | Giorgio Metafuno | Lecture 1: Operator semigroups methods in evolution equations: an introduction. |
| 17:30 - 18:30 | | Transport to the Rectorate and coffee break |
| 18:30 - 20:30 | | Visit to Lecce |
| 21:00 | | Dinner |

Wednesday 19 April

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| 09:00 - 11:00 | Gianfausto Salvadori | Lecture 2: Multivariate modeling via Copulas: theory and practice |
| 11:00 - 11:30 | | Coffee Break |
| 11:30 - 12:30 | Adriano Barra | Symmetry breaking and Andronov-Hopf bifurcation: analogies between modus operandi in Analytical Mechanics and Statistical Mechanics |
| 12:30 - 14:30 | | Lunch |
| 14:30 - 15:00 | | Transport to the Dept. of Mathematics |
| 15:00 - 17:00 | Giorgio Metafuno | Lecture 2: Operator semigroups methods in evolution equations: an introduction. |
| 17:00 - 18:00 | | Transport to the Rectorate and coffee break |
| 18:00 - 20:00 | | Projects |
| 21:00 | | Dinner |

Thursday 20 April

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| 09:00 - 11:00 | Gianfausto Salvadori | Lecture 3: Multivariate modeling via Copulas: theory and practice |
| 11:00 - 11:30 | | Coffee Break |
| 11:30 - 12:30 | Elena Agliari | Criticality in Graphs |
| 12:30 - 14:30 | | Lunch |
| 14:30 - 15:00 | | Transport to the Dept. of Mathematics |
| 15:00 - 17:00 | Giorgio Metafuno | Lecture 3: Operator semigroups methods in evolution equations: an introduction. |
| 17:00 - 18:00 | | Transport to the Rectorate and coffee break |
| 18:00 - 20:00 | | Projects |
| 21:00 | | Dinner |

Friday 21 April

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|---------------|----------------------|--|
| 09:00 - 11:00 | Gianfausto Salvadori | Lecture 4: Multivariate modeling via Copulas: theory and practice |
| 11:00 - 11:30 | | Coffee Break |
| 11:30 - 12:30 | Elisabetta Mangino | Is linear chaos a "quiet" chaos? |
| 12:30 - 14:30 | | Lunch |
| 14:30 - 15:00 | | Transport to the Dept. of Mathematics |
| 15:00 - 17:00 | Giorgio Metafuno | Lecture 4: Operator semigroups methods in evolution equations: an introduction. |
| 17:00 - 18:00 | | Transport to the Rectorate and coffee break |
| 18:00 - 20:00 | | Projects |
| 21:00 | | Dinner |

Saturday 22 April

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| 09:00 - 11:00 | Projects presentation. |
| 11:00 - 20:30 | Excursion |
| 21:00 | Dinner |

Abstracts of the courses

Operator semigroups methods in evolution equations: an introduction

Giorgio Metafuno (University of Salento)

List of topics:

- The exponential function of a bounded operator. Basic semigroup theory, motivations and elementary properties: generator, resolvent, differential equation. Basic spectral theory of closed operators.
- The theorem of Hille-Yosida. Dissipativity and contraction semigroups.
- Analytic semigroups.
- Semigroups generated by second order elliptic differential operators. Schrödinger semigroups.

Multivariate modeling via Copulas: theory and practice

Gianfausto Salvadori (University of Salento)

The random dynamics of environmental phenomena, as well as the ones of economic/financial processes, are typically ruled by a number of non-independent variables, i.e. the behavior is multivariate. The Theory of Copulas provides, on the one hand, a valuable tool to explore and understand the (random) interactions between the variables at play, and, on the other hand, gives the possibility to model and simulate the joint dynamics of the variables of interest. The course will outline the basic features of Copulas, and practical examples will be used to illustrate how Copulas can be used in applications.

List of topics:

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|---|--|
| • Introduction to copulas: definitions and properties | • Measures of association (Kendall, Spearman, Blomqvist) |
| • Sklar's Theorem | • Conditional mixtures |
| • Invariance property | • Extra-parametrization techniques |
| • Conditional probabilities | • Multivariate return period |
| • Principles of multivariate simulation | • Multivariate design |
| • Survival copulas | • Dynamic return period |
| • Empirical copulas | • Failure probability |
| • Extreme Value copulas | • Structural risk |
| | • Ties and randomization |

Abstracts of the talks

Criticality in Graphs

Elena Agliari (Sapienza University of Rome)

In the first part of the talk we will provide a basic introduction to graph theory, reviewing the main definitions, concepts and models. In the second part we will discuss critical phenomena in graphs from several perspectives: from percolation (where, as we progressively dilute the graph edges/nodes, the topological properties of the graph can exhibit abrupt changes) to statistical mechanics (where the behavior of interacting units living on the graph nodes can exhibit spontaneous collective features according to the topology of the interaction pattern).

Symmetry breaking and Andronov-Hopf bifurcation: analogies between modus operandi in Analytical Mechanics and Statistical Mechanics

Adriano Barra (University of Salento)

In this talk I aim to highlight conceptual and formal analogies between some of the key mathematical scaffolds of analytical mechanics and statistical mechanics. Starting from the statistical mechanics side, I will try to revise the simplest toy-model that shows critical behaviour (namely the mean field Ising model): following standard approaches, we will construct an explicit expression for its free energy to extremize w.r.t. its natural order parameter -the magnetization- and, in this way, we will obtain the model's phase diagram. This result will be achieved through two statistical mechanics routes [3]: the former more useful to check every mathematical passage, the latter best representing the underlying physics. Once properly explored the statistical mechanical side, I will show how it is possible to recover the whole picture we painted even without any knowledge of the discipline. In particular we will see how the free energy can play as the principal Hamilton function in the space of its tunable parameters: assuming the latter to be the one-body & two-body couplings -as standard- in this equivalence they will play as "generalised space" and "generalised time". This allows solving the model via the Hamilton-Jacobi approach (typical of analytical mechanics) in this 1+1 dimensional space. Through this route we will see that equivalences are several: the spatial streaming of the Hamilton-Jacobi equation for the free energy returns a Burgers equation for its spatial derivative, i.e. the magnetization (that plays the role of the velocity in this analogy). In the thermodynamic limit such equation becomes inviscid, hence reducing to a Riemann-Hopf type that is the standard tool to see the genesis of a Andronov-Hopf bifurcation: crucially we will see that such a bifurcation happens exactly where the system experiences critical behaviour in the statistical mechanics counterpart [2]. Finally will check that everything keeps holding even if we move to the "continuous version" (where the microscopic variables are no longer binary spins), i.e. the Van Der Waals theory [1].

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- [1]. A. Barra, A. Moro, Exact solution of the van der Waals model in the critical region, *Annals of Physics* 359, 290-299 (2015). paper
- [2]. A. Barra, A. Di Lorenzo, F. Guerra, A. Moro, On quantum and relativistic mechanical analogues in mean field spin models, *Proceedings of the Royal Society A (London)*, 470, 20140589 (2014) paper
- [3]. A. Barra, The Mean Field Ising Model through Interpolating Techniques, *Journal of Statistical Physics* 132, 5, 787-809, (2008). paper

Is linear chaos a "quiet" chaos?

Elisabetta Mangino (University of Salento)

The talk is meant as an introduction to linear transitivity and linear chaos in infinite-dimensional Banach spaces, through motivating examples and comparisons with the finite-dimensional case.

Spectral Theory for Nonautonomous Differential Equations

Martin Rasmussen (Imperial College London)

This talk is an introduction to the spectral theory for nonautonomous differential equations. I will explain four different types of spectra: the Lyapunov spectrum, the Sacker-Sell spectrum, the Morse spectrum and the Bohl spectrum. While both the Lyapunov and the Bohl spectrum are spectral concepts taking individual solutions into account, the Sacker-Sell and the Morse spectrum work with nontrivial decompositions of the space into finitely many components. Differences and similarities between the different types of spectra will be highlighted, and applications will be discussed.

Parts of this talk are joint work with Fritz Colonius (University of Augsburg), Thai Son Doan (Vietnam Academy of Science and Technology), Peter Kloeden (Huazhong University of Science and Technology), and Ken Palmer (National Taiwan University).