Chaos: why, where and how much

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We will consider the chaos which appears in reals problems, either in sciences or in technology, assuming, for concreteness the following facts: a) it is admissible that the phenomenon to be studied is deterministic; b) one has the equations describing the evolution of the system, and c) comparing with experiments (physical and not numerical, of course) the predictions following from the model are sufficiently good. Both continuous and discrete systems will be considered.

The system, despite satisfying the previous conditions, can loose predictability or, at least, the time of validity of the predictions can be short. The lack of predictability is mainly due to the existence of hyperbolic (perhaps in a weak or partial sense) invariant objects in the phase space.

The problems appear due to the fact that some invariant manifolds, which in a system with regular behavior coincide, become transversal in the realistic system. Hence one should understand the lack of coincidence of manifolds, a phenomenon usually known as splitting of manifolds. Analytical tools to compute the splitting in simple models and symbolic/numerical tools to compute invariant manifolds will be introduced. This is a key tool to detect transversal (or weakly transversal) homoclinic orbits and chains of heteroclinics orbits.

This will allow to locate the chaotic zones. With this goal in mind, some simple models for return maps will be presented. Then we shall discuss some paradigmatic models both in conservative and in dissipative systems. In particular commenting about Hamiltonian chaos and about strange attractors. Depending on the available time we shall comment, also, on the apparition of chaos in PDE.

Creation and destruction of some chaos is related to global bifurcations, due to the tangential homoclinic and heteroclinic orbits.

Another key point is to try to predict how much chaos appears in a given system. Perhaps the size is so small that its effects can be neglected. Beyond theoretical tools in the case of small perturbations of a regular system, some standard numerical tools will be presented, like the computation of finite time Lyapunov exponents and the mean exponential growth of nearby orbits (MEGNO). One can also comment on some degenerate chaotic systems which, in contrast with the typical behavior, have zero Lyapunov exponent.

For the full course it is planned to insist in that the explanation of the observed behavior is based in the geometry of the objects which appear in the phase space.