CRITICS Summer Meeting 2017

Valladolid 4 - 15 September

Scientific Committee:
Henk Dijkstra
Carmen Núñez
Rafael Obaya
Martin Rasmussen
Alexander Skupin

Organizing Committee:
Iacopo P. Longo
Ismael Maroto
Sylvia Novo
Carmen Núñez
Rafael Obaya
Ana M. Sanz

artwork by: @iacopolongo

MARIE CURIE ACTIONS
CRITICS
Critical Transitions in Complex Systems
European Commission
ESCUELA DE INGENIERÍAS INDUSTRIALES
imUVa
Instituto de Matemáticas
GOBIERNO DE ESPAÑA
MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD

SUMMER SCHOOL, September 4 – 8

	M	Tu	W	Th	F
9:30 – 11:30	**Skupin**	**Lucarini**	**Simó**	**Simó**	**Lamb**
12:00 – 13:00	**Skupin**	**Lucarini**	**Sargent**	**Lamb**	**Lamb**
15:30 – 16:30	**Skupin**	**Simó**		**Lamb**	
16:30 – 17:30	**Sargent**	**Simó**		**Lamb**	

Registration: Monday 4, 9:00-9:20, Escuela de Ingenierías Industriales, Seminario de Matemáticas (second floor)
Lectures: Escuela de Ingenierías Industriales, Room 2.1 (second floor)
Coffee breaks: 11:30 – 12:00, Bar Caribe
Lunches: Served at 13:30, Hotel Olid (20 minutes from the Escuela)

Two project works will be proposed respectively by Professors Sargent and Skupin. Students will be divided in two groups, each one focused on one of the two projects. Exercises on the content of the other courses will be assigned too. The afternoons of Wednesday and Friday, and the whole Saturday will be devoted to such work.

EXCURSION, Sunday 10

Those participants who have registered for the excursion will visit Salamanca on Sunday.

Departure: Hotel Olid, 9:30. There will be a bus at the hotel entrance.
Please be at the entrance strictly on time.

Return: Hotel Olid, 19:15 (approximately).

In Salamanca a local guide will show you the old city, the Cathedrals, and Ieronimus (an amazing walk through the interior and exterior of the towers of the Cathedrals).

Lunch will be in Restaurante El Bardo, in Calle Compañía 8, around 14:30.

SUMMER SCHOOL – Participants

Sajjad BAKRANI BALANI, Imperial College London
Daniele CASTELLANA, Utrecht University
Gabriel FUHRMANN, Friedrich Schiller University Jena
Raphael GERLACH, Paderborn University
Boumediene HAMZI, AlFaisal University
Michael HARTL, Imperial College London
Jeroen LAMB, Imperial College London
Johannes LOHMANN, University of Copenhagen
Iacopo Paolo LONGO, Universidad de Valladolid
Valerio LUCARINI, University of Reading
Stefano MAGNI, University of Luxembourg
Ismael MAROTO CAMARENA, Universidad de Valladolid
Holger METZLER, Max Planck Institute for Biogeochemistry, Jena
M. Usman MIRZA, Wageningen University
Moussa NDOUR, TU Dresden
Sylvia NOVO, Universidad de Valladolid
Carmen NÚÑEZ, Universidad de Valladolid
Karl NYMAN, University of Copenhagen
Rafael OBAYA, Universidad de Valladolid
Guillermo OLICÓN MÉNDEZ, Imperial College London
Christian PANGERL, Imperial College London
Lucía PÉREZ, University of Oviedo
Courtney QUINN, University of Exeter
Flavia REMO, Friedrich Schiller University Jena
Pablo RODRÍGUEZ-SÁNCHEZ, Wageningen University
Cristina SARGENT, Imperial College London
Carles SIMÓ, Universitat de Barcelona
Alexander SKUPIN, University of Luxembourg
Damian SMUG, University of Exeter
Kalle TIMPERI, Imperial College London
Chun XIE, University College Cork

WORKSHOP, September 11 – 15

Monday 11

09:30 – 10:00: Registration
10:05 – 10:10: Opening
10:45 – 11:30: Cristina Masoller

12:00 – 12:45: Luonan Chen
12:45 – 13:30: Ulrike Feudel

15:50 – 16:30: Sebastian Wieczorek
16:30 – 17:10: Anna Vanselow
17:10 – 17:50: Moussa Ndour

20:30: Vino español (informal dinner) at Hotel Olid

Tuesday 12

10:00 – 10:45: Nick Jones
10:45 – 11:30: Frank Kwasniok

12:00 – 12:40: Peter Ashwin
12:40 – 13:20: Paul Ritchie

15:50 – 16:30 Ilya Pavlyukevich
16:30 – 17:10 Chun Xie

17:30 – 19:00: Guided visit to the Museo de Escultura San Gregorio

Wednesday 13

10:00 – 10:45: Dmitry Turaev
10:45 – 11:30: Alan Hastings

12:00 – 12:40: Kathrin Padberg-Gehle
12:40 – 13:20: Peter Giesl

15:50 – 16:30: Pablo Rodríguez-Sánchez
16:30 – 18:30: Posters session

21:00: Conference dinner, Restaurante La Parrilla de San Lorenzo

Thursday 14

10:00 – 10:45: Mickael Chekroun
10:45 – 11:30: Rafael Obaya

12:00 – 12:40: Iacopo P. Longo
12:40 – 13:20: Lucía Pérez

15:50 – 17:50: Presentation of the Summer School projects

Friday 15

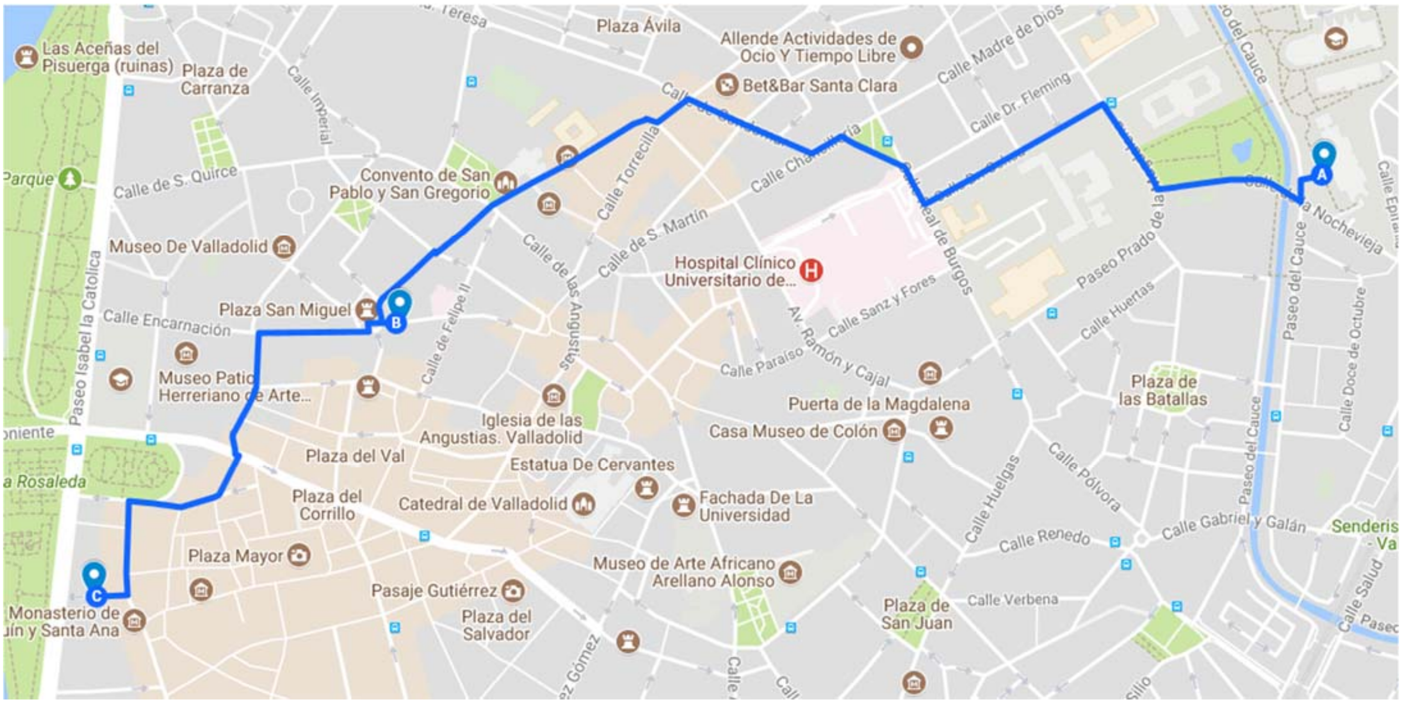
10:00 – 10:45: Roberto Barrio
10:45 – 11:30: Jens Starke
12:00 – 12:45: Àngel Jorba

Registration:	Monday 4, 9:00-9:20, Escuela de Ingenierías Industriales, Seminario de Matemáticas (second floor)
Talks:	Escuela de Ingenierías Industriales, Room 2.1 (second floor)
Coffee breaks:	11:30 – 12:00, Cafetería de la Escuela (basement)
Lunches:	Served at 13:45, Hotel Olid (20 minutes from the venue)

WORKSHOP – Participants

Peter ASHWIN, University of Exeter
Sajjad BAKRANI BALANI, Imperial College London
Roberto BARRIO, Universidad de Zaragoza
Daniele CASTELLANA, Utrecht University
Mickael CHEKROUN, University of California, Los Angeles
Luonan CHEN, Shanghai Institute for Biological Sciences
Anna Maria CHERUBINI, University of Salento
Ulrike FEUDEL, Universität Oldenburg
Jan GAIRING, Universität München
Raphael GERLACH, Paderborn University
Peter GIESL, University of Sussex
Boumediene HAMZI, AlFaisal University
Michael HARTL, Imperial College London
Alan HASTINGS, University of California, Davis
Tobias JÄGER, Friedrich Schiller University Jena
Nick JONES, Imperial College London
Ángel JORBA, Universitat de Barcelona
Frank KWASNIOK, University of Exeter
Jeroen LAMB, Imperial College London
Johannes LOHMANN, University of Copenhagen
Iacopo Paolo LONGO, Universidad de Valladolid
Ismael MAROTO CAMARENA, Universidad de Valladolid
Cristina MASOLLER Universitat Politècnica de Catalunya
Holger METZLER, Max Planck Institute for Biogeochemistry, Jena
M. Usman MIRZA, Wageningen University
Moussa NDOUR, TU Dresden
Sylvia NOVO, Universidad de Valladolid
Carmen NÚÑEZ, Universidad de Valladolid
Karl NYMAN, University of Copenhagen
Rafael OBAYA, Universidad de Valladolid
Guillermo OLICÓN MÉNDEZ, Imperial College London
Kathrin PADBERG-GEHLE, Leuphana University Lueneburg

Christian PANGERL, Imperial College London
Ilya PAVLYUKEVICH, Friedrich Schiller University Jena
Lucía PÉREZ, University of Oviedo
Courtney QUINN, University of Exeter
Martin RASMUSSEN, Imperial College London
Flavia REMO, Friedrich Schiller University Jena
Paul RITCHIE, University of Exeter
Pablo RODRÍGUEZ-SÁNCHEZ, Wageningen University
Ana María SANZ, Universidad de Valladolid
Cristina SARGENT, Imperial College London
Damian SMUG, University of Exeter
Jens STARKE, University of Rostock
Kalle TIMPERI, Imperial College London
Dmitry TURAEV, Imperial College London
Egbert VAN NES, Wageningen University
Anna VANSELOW, University of Oldenburg
Sebastian WIECZOREK, University College Cork
Ke WU, ETH Zurich
Chun XIE, University College Cork



A: Escuela de Ingenierías: venue. Paseo del Cauce 5
B: Hotel Olid: accommodation, lunches, and vino español (11th). Plaza de San Miguel 10
C: Restaurante La Parrilla de San Lorenzo: conference dinner (13th), Calle Pedro Niño 1

INTERNET ACCESS

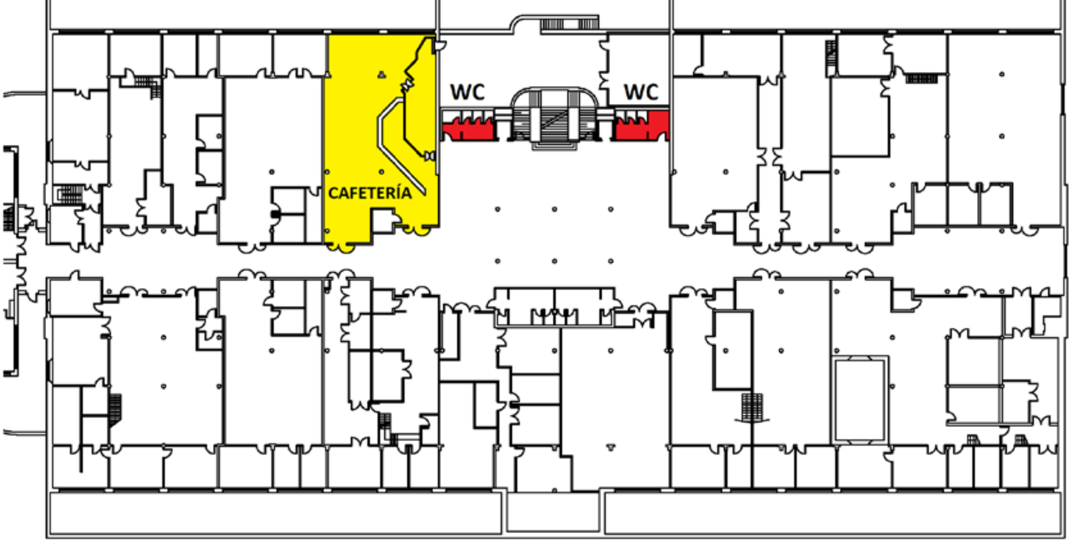
All participants may access the Internet and related services in three different ways:

1. By means of the computers in the Universia rooms (ground floor of the E.I.I. see **COMPUTERS** in the map). All of the computers in these rooms are available for the participants in the workshop. The only requirement is to obtain an anonymous UVA card at the porter's office (see **P** in the map); there you will be required to leave a card or passport in order to guarantee the return of the UVA card.

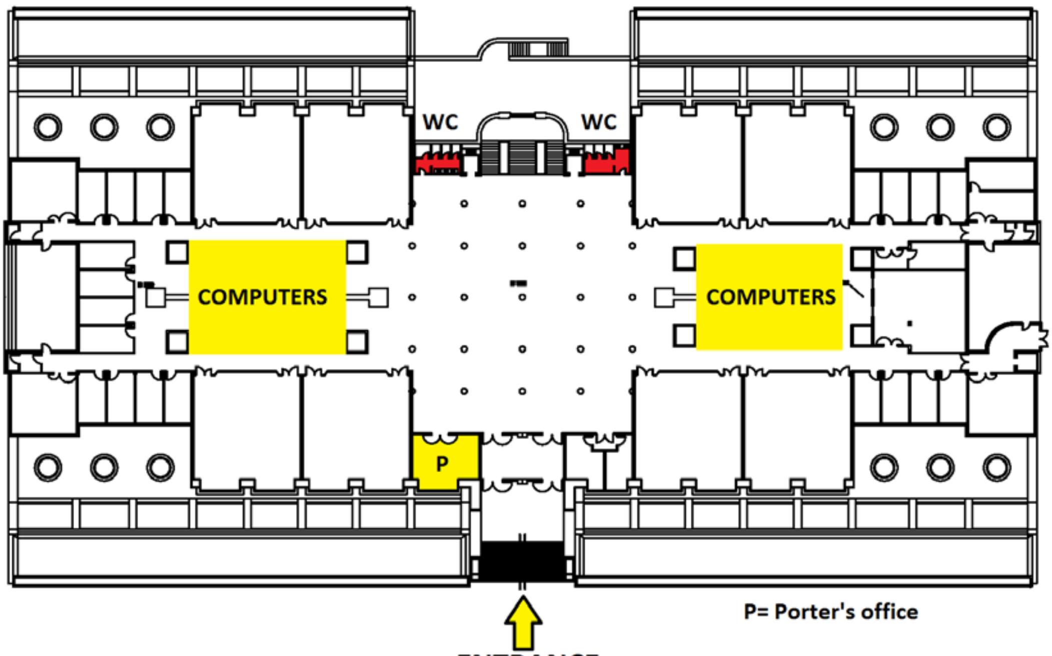
2. By Wi-Fi. User: Critics2017
   Password: CHAOS2017

3. Those having an identity granting their access to the European Education Roaming Network (also known as EDUROAM) may access this network from the E.I.I. or elsewhere within the University campus. In order to do so, the EDUROAM software must be set up so that the Configuration box remains unchecked and that the field Username is of the form name@domain.country.

BASEMENT

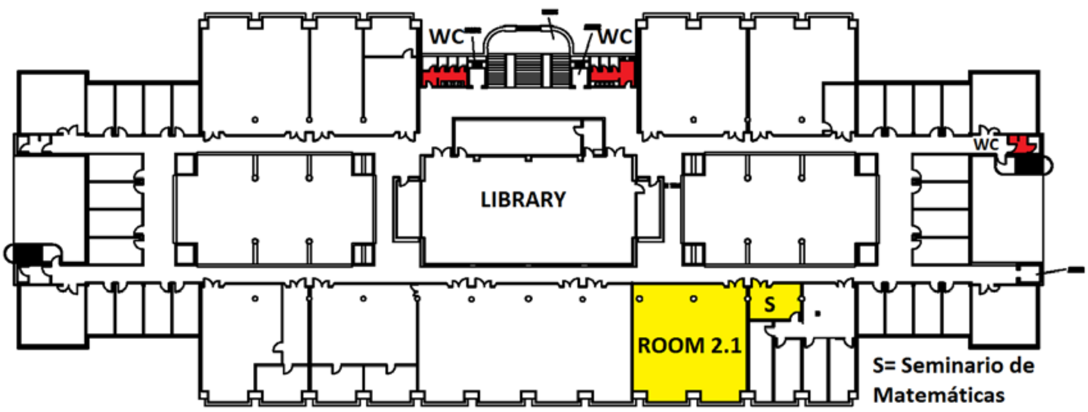


GROUND FLOOR



P= Porter's office

SECOND FLOOR



S= Seminario de Matemáticas

Summer School: Abstracts

Single Cell Analysis to Dissect Dynamics of Cellular Heterogeneity in Health and Disease

Alexander Skupin, Luxembourg Centre for Systems Biomedicine

Cell fate commitment is the key process for understanding how a multicellular organism's genotype gives rise to phenotypic traits including disease development and progression. Recent experimental advancements in single cell RNAseq approaches allow now measuring a cell's state by its transcriptional state defined by the abundance of the versatile mRNAs.

In the lecture, I will give a brief introduction into basic molecular biology and demonstrate how we can combine live cell imaging and single cell omics approaches with mathematical analysis to identify molecular mechanisms and general characteristics of cell fate dynamics.

For this purpose we apply imaging-based approaches to toxin induced mitochondrial dysfunction to reveal potential early warning signals in Parkinson's disease und characterize underlying principles. To understand cell fate and differentiation dynamics, we derived a new correlation based measure for single cell transcriptomics time course data that allows for population classification during differentiation and validated this approach in blood cell differentiation and carcinogenesis. Furthermore, the course will further exemplify how single cell analysis and methods from non-linear time course analysis opens new routes for individualized treatments and can be used to investigate mutation specific modifications of cellular regulation.

Finally, we will have a hands-on session where we will study typical single cell RNAseq data and perform the first steps of the corresponding analysis challenges using R.
(For this part interested participants should bring their laptop with R version 3.3 or later and Monocle 2.4 installed,
see also: https://bioconductor.org/packages/release/bioc/html/monocle.html.)

Response and Fluctuations in the Climate Systems

Valerio Lucarini, University of Reading and University of Hamburg

The climate is a complex, chaotic, non-equilibrium system featuring a limited horizon of predictability, variability on a vast range of temporal and spatial scales, instabilities resulting into energy transformations, and mixing and dissipative processes resulting into entropy production. Despite great progresses, we still do not have a complete theory of climate dynamics able to encompass instabilities, equilibration processes, fluctuations, and response to changing parameters of the system. The lecture will be divided roughly in three parts.

- First, we will provide an overview of the large-scale dynamical processes inside the climate system taking the point of view of nonequilibrium thermodynamics. We will introduce concepts like efficiency, irreversibility, and entropy production and investigate their relevance for understanding the properties of the climate system. We will show, in particular, how they can be relevant for understanding critical transitions and for interpreting circulations in planetary circulations different from the Earth's [1].

- Second, we will introduce the statistical mechanical point of view on nonequilibrium systems. We will present some applications of the response theory developed by Ruelle for non-equilibrium systems, showing how it allows for setting on firm ground and on a coherent framework concepts like climate sensitivity and climate response. We will show results for simple, intermediate complexity, and comprehensive global climate models. The results are promising in terms of suggesting new ways for approaching the problem of climate change prediction and for using more efficiently the enormous amounts of data produced by modeling groups around the world [2].

- Third, we will delve into the problem of studying the response of the system when we have rough dependence of the system's properties on the parameters. We will discussion a paradigm for "climatic surprises", i.e. occurrence of radically new climate phenomena resulting from perturbations. The lack of correspondence between forced and free fluctuations shows a clear violation of the fluctuation-dissipation theorem [3]. Finally, we will provide a new point of view on critical transitions based on the concept of edge state, which is the gate for noise- induced transitions. We will introduce the Melancholia states, which are climate repellors separating the co-existing warm and snowball states of the climate system [4].

References

[1] V. Lucarini, R. Blender, C. Herbert, F. Ragone, S. Pascale, and J. Wouters, Mathematical and Physical Ideas for Climate Science, Reviews of Geophysics 52, 809-859 (2014)

[2] V. Lucarini, F. Ragone, and F. Lunkeit, Predicting Climate Change Using Response Theory: Global Averages and Spatial Patterns, Journal of Statistical Physics 166, 1036–1064 (2017)

[3] A. Gritsun and V. Lucarini, Fluctuations, Response, and Resonances in a Simple Atmospheric Model, Physica D 349, 62-76 (2017)

[4] V. Lucarini and T. Bodai, Edge states in the climate system: exploring global instabilities and critical transitions Nonlinearity 30 R32 (2017)

Chaos: why, where and how much

Carles Simó, Universitat de Barcelona

We will consider the chaos which appears in reals problems, either in sciences or in technology, assuming, for concreteness the following facts: a) it is admissible that the phenomenon to be studied is deterministic; b) one has the equations describing the evolution of the system, and c) comparing with experiments (physical and not numerical, of course) the predictions following from the model are sufficiently good. Both continuous and discrete systems will be considered.

The system, despite satisfying the previous conditions, can loose predictability or, at least, the time of validity of the predictions can be short. The lack of predictability is mainly due to the existence of hyperbolic (perhaps in a weak or partial sense) invariant objects in the phase space.

The problems appear due to the fact that some invariant manifolds, which in a system with regular behavior coincide, become transversal in the realistic system. Hence one should understand the lack of coincidence of manifolds, a phenomenon usually known as splitting of manifolds. Analytical tools to compute the splitting in simple models and symbolic/numerical tools to compute invariant manifolds will be introduced. This is a key tool to detect transversal (or weakly transversal) homoclinic orbits and chains of heteroclinics orbits.

This will allow to locate the chaotic zones. With this goal in mind, some simple models for return maps will be presented. Then we shall discuss some paradigmatic models both in conservative and in dissipative systems. In particular commenting about Hamiltonian chaos and about strange attractors. Depending on the available time we shall comment, also, on the apparition of chaos in PDE.

Creation and destruction of some chaos is related to global bifurcations, due to the tangential homoclinic and heteroclinic orbits.

Another key point is to try to predict how much chaos appears in a given system. Perhaps the size is so small that its effects can be neglected. Beyond theoretical tools in the case of small perturbations of a regular system, some standard numerical tools will be presented, like the computation of finite time Lyapunov exponents and the mean exponential growth of nearby orbits (MEGNO). One can also comment on some degenerate chaotic systems which, in contrast with the typical behavior, have zero Lyapunov exponent.

For the full course it is planned to insist in that the explanation of the observed behavior is based in the geometry of the objects which appear in the phase space.

Short course on Random Dynamical Systems

Jeroen S.W. Lamb, Imperial College London

Dynamical systems describe the time-evolution of variables that characterize the state of a system. In deterministic autonomous dynamical systems, the corresponding equations of motion are independent of time and constant, but in random dynamical systems the equations of motion explicitly depend on a stochastic process or random variable.

The development of the field of deterministic dynamical systems – including “chaos” theory - has been one of the scientific revolutions of the twentieth century, originating with the pioneering insights of Poincaré, providing a geometric qualitative understanding of dynamical processes, aiding and complementing analytical and quantitative viewpoints.

Motivated by increasing demands on modelling from scientists, during the last decade there has been an increasing interest in time-dependent and in particular random dynamical systems, often – but not exclusively - described by stochastic differential equations. Despite the obvious scientific importance of the field, with applications ranging from physics and engineering to bio-medical and social sciences, a geometric qualitative theory for random dynamical systems is still in its infancy.

This short course consists of an introduction to random dynamical systems, from a predominantly geometric point of view. The aim is to introduce basic concepts in the context of simple examples. We discuss some elementary results and highlight open questions.

The course is aimed at graduate students; some elementary background in dynamical systems and probability theory is useful, but not a strict prerequisite.

Outline:
1. Barnsley’s “chaos game” as a random dynamical system.
2. Random circle maps: Lyapunov exponents and synchronisation.
3. Set-valued dynamical systems describing the aggregate behaviour of random dynamical systems with bounded noise.
4. Early warning signals for bifurcations in random dynamical systems.

Mathematics in Innovation – Study group

Cristina Sargent, Imperial College London

During this study group, we will address real-life problems using complex dynamical systems concepts and methods. We will learn about developing solutions which are feasible for implementation in practice.

The program will start with the presentation of one or two problems (details to follow) together with relevant background information. The problems are concerned with real systems which undergo critical transitions. The question is how to create tools which could be used to investigate possible scenarios, predict sudden shifts in useful time and advise on appropriate measures to be taken.

The group will collaborate to develop an innovation roadmap from the latest research to an algorithm which uses several data sets. We do not expect to resolve all the stages of this innovation cycle in this study group. Depending on time availability we may also consider what would constitute a proof of concept and ways to validate a good solution.

The study group activities will conclude with a presentation during the second week, where the participants will present their work, the analysis of the problems, insights achieved and ideas for further work.

Workshop: Abstracts

INVITED TALKS

Roberto BARRIO, Universidad de Zaragoza

Pattern bifurcations of isolated and coupled neurons and control strategies via global stimuli

The study of the synchronization patterns of small neuron networks that control several biological processes has become an interesting growing discipline. Some of these synchronization patterns of individual neurons are related with some undesirable neurologic diseases, and they are believed to play a crucial role in the emergence of pathological rhythmic brain activity in different diseases, like Parkinson's disease. We show, on 3-cell Central Pattern Generators, how with a suitable combination of short and weak global inhibitory and excitatory stimuli over the network, we can switch between different stable bursting patterns in small neuron networks (in our case a 3-neuron network). We develop a systematic study showing and explaining the effects of applying the pulses at different moments. Moreover, we apply the technique on a completely symmetric network and on a slightly perturbed one (a more realistic situation). The approach of using global stimuli, as in the case of applying a current or a chemical substance to all the network, allows one to avoid undesirable synchronization patterns with nonaggressive stimuli. The control technique takes advantage of the information given by detailed biparametric “roadmaps'”. Such a roadmap provides an exhaustive information about the dynamics of a single neuron that one must have in order to build small neuron networks and to study rhythmogenesis in central pattern generators (CPG). Also, the use of the roadmaps reveals the existence of heteroclinic cycles between saddle fixed points (FP) and invariant circles (IC) in a 3-cell CPG network (leech heart neurons). Such a cycle underlies a robust “jiggling'' behavior in bursting synchronization.

Mickaël CHEKROUN, University of California Los Angeles

Closures for Stochastic Partial Differential Equations Driven by Degenerate Noise

Stochastic partial differential equations (SPDEs), with bilinear drift and driven by a degenerate additive noise, will be considered. For such equations, we will present new analytic formulas for the parameterizations of the scales lying beyond a cutoff wavenumber. The derivation of these formulas takes place within a variational approach relying on the theory of stochastic parameterizing manifolds whose main tools and concepts will be introduced. The relationships with the ergodic theory of SPDEs will be discussed and applications to closure in the context of “Burgulence” will be presented. The role of path-dependent, non-Markovian coefficients arising in the related closure systems will be also discussed.

Joint work with Honghu Liu, James C. McWilliams, and Shouhong Wang.

Luonan CHEN, Shanghai Inst. for Biological Sc. & Chinese Acad. of Sciences

Hunt for the tipping points of biological processes and complex diseases by dynamic network biomarkers based on the observed data

Considerable evidence suggests that during the progression of complex diseases, the deteriorations are not necessarily smooth but are abrupt, and may cause a critical transition from one state to another at a tipping point. Here, we develop a model-free method to detect early-warning signals of such critical transitions or the tipping point, even with only a small number of samples (or one sample). Specifically, we theoretically derive criteria based on a dynamic network biomarker (DNB) that serves as a general early-warning signal indicating an imminent sudden deterioration before the critical transition occurs. Based on theoretical analyses, we show that predicting the tipping point from small samples and strong noises is achievable provided that there are a large number of measurements for each sample, e.g., high-throughput data (omics data). We employ gene expression data of many diseases to demonstrate the effectiveness of our method. The relevance of DNBs with the diseases was also validated by related experimental data (e.g., liver cancer, lung injury, influenza, type-2 diabetes) and functional analysis. DNB can also be used for the analysis of nonlinear biological processes, e.g., cell differentiation process. In addition, DNB is further extended to DNM for tipping point prediction of general nonlinear systems.

Ulrike FEUDEL, Carl von Ossietzky University Oldenburg, Germany

Death and revival of chaos

We investigate the death and revival of chaos under the impact of a monotonous time-dependent forcing that changes its strength with a non-negligible rate. Starting on a chaotic attractor it is found that the complexity of the dynamics remains very pronounced even when the driving amplitude has decayed to rather small values. When after the death of chaos the strength of the forcing is increased again with the same rate of change, chaos is found to revive but with a different history. This leads to the appearance of a hysteresis in the complexity of the dynamics. To characterize these dynamics, the concept of snapshot attractors is used, and the corresponding ensemble approach proves to be superior to a single trajectory description, that turns out to be non-representative. The death (revival) of chaos is manifested in a drop (jump) of the standard deviation of one of the phase-space coordinates of the ensemble; the details of this chaos-nonchaos transition depend on the ratio of the characteristic times of the amplitude change and of the internal dynamics. It is demonstrated that chaos cannot die out as long as underlying transient chaos is present in the parameter space. As a condition for a “quasistatically slow” switch-off, we derive an inequality which cannot be fulfilled in practice over extended parameter ranges where transient chaos is present. These observations need to be taken into account when discussing the implications of “climate change scenarios” in any nonlinear dynamical system.

Joint work with Balint Kaszas and Tamas Tel, Eötvös University Budapest, Hungary

References:

B. Kaszas, U. Feudel, and T. Tel, Death and revival of chaos, Phys. Rev. E **94**, 062221 (2016).

Alan HASTINGS, University of California Davis

Tipping points and early warning signals in ecological systems

I will focus both on specific ecological systems and on models that arise in ecology that exhibit bifurcations, discussing the potential for early warning signals and how these signals could possibly be used in management. The specific systems of interest will be coral-algal-grazer systems and I will also point out that features of these systems are present in other ecological systems. I will then turn to discussing the challenges of finding signals in more complex ecological systems that either have more species, more complex dynamics, or are distributed over space.

Nick JONES, Imperial College London

Highly Comparative Time Series Analysis

I will discuss our work assembling a giant library of 7000 time series estimators and 35,000 empirical and model generated signals. I'll discuss our efforts to automatically contextualize new estimators and new data and to identify estimators which are generically useful for diverse classification tasks; I'll also give classification and regression examples with biomedical relevance. I'll touch on our efforts to automatically identify the dimensionality of unobserved parametric variation in an unlabelled set of time series and point to the relevance of our approach for identifying anomalies and early warnings.

Àngel JORBA, Universitat de Barcelona

High order approximation of invariant manifolds of fixed point of Poincare maps

In this talk we will explain a computational technique (called jet transport) to compute high order derivatives of Poincare maps with respect to initial data and/or parameters. The method is based on the use of automatic differentiation techniques on the propagation of the flow of the ODE. If the flow propagation is done by means of a Taylor method, the resulting procedure is efficient enough to be carried out even with extended precision arithmetic. As applications, we mention the effective computation of normal forms, centre manifolds and stable/unstable manifolds of periodic orbits. In the talk we will discuss the effective computation of 1D and 2D stable/unstable manifolds of periodic orbits on concrete examples.

This presentation summarises collaborations with A. Farres, G. Gimeno, M. Jorba-Cusco, N. Miguel and M. Zou.

Frank KWASNIOK, University of Exeter

Detecting, anticipating and predicting critical transitions using data-driven nonstationary dynamical modelling

The talk discusses detection, anticipation and prediction of critical transitions from time series. A nonstationary stochastic dynamical model of appropriate complexity to capture the transition mechanism under consideration is estimated from data. In the simplest case, the model is a one-dimensional effective Langevin equation, but also higher-dimensional reconstructions based on time-delay embedding and local modelling are considered. Moreover, spatially extended systems are also discussed. The nonstationary models are extrapolated beyond the learning data window to predict the nature and timing of critical transitions. The methods are first demonstrated on simulated data from known mathematical systems and then applied to critical transitions in the field of climate science, e.g., arctic sea ice and the meridional overturning circulation.

Cristina MASOLLER, Universitat Politècnica de Catalunya

Identifying and characterizing regime transitions with network-based data analysis tools

Complex systems can undergo gradual or sudden transitions between different dynamical regimes. While some transitions can be safe, others can be catastrophic (such as desertification, population extinctions, arrhythmias, etc.). It these situations it is important to anticipate transitions by identifying appropriated early-warning signals, and a lot of efforts are nowadays focused on developing reliable diagnostic tools that can be directly applied to observed data. In this presentation I will discuss our recent work where we have employed various network tools (correlation networks, symbolic networks, horizontal visibility) for identifying and predicting regime transitions in different complex systems.

References:

[1] G. Tirabassi, J. Viebahn, V. Dakos, H. A. Dijkstra, C. Masoller, M. Rietkerk, and S.C. Dekker, "*Interaction network based early-warning indicators of vegetation transitions*", Ecological Complexity 19, 148 (2014).

[2] C. Masoller, Y. Hong, S. Ayad, F. Gustave, S. Barland, A. J. Pons, S. Gomez, and A. Arenas, "*Quantifying sudden changes in dynamical systems using symbolic networks*", New Journal of Physics 17, 023068 (2015).

[3] A. Aragoneses, L. Carpi, N. Tarasov, D. V. Churkin, M. C. Torrent, C. Masoller, and S. K. Turitsyn, "*Unveiling temporal correlations characteristic to phase transition in the intensity of fibre laser radiation*", Phys. Rev. Lett. 116, 033902 (2016).

[4] T. A. Schieber, L. Carpi, A. Diaz-Guilera, P. M. Pardalos, C. Masoller and M. G. Ravetti, "*Quantification of network structural dissimilarities*", Nat. Comm. 8, 13928 (2017).

Rafael Obaya, Universidad de Valladolid

Some discontinuous bifurcation scenarios in nonautonomous reaction-diffusion equations.

The aim of this talk is to explain some discontinuous bifurcation scenarios which appear in the nonautonomous dynamical systems induced by certain families of scalar dissipative time-dependent parabolic PDEs with Neumann or Robin boundary conditions. First, we focus on the multiplicity and structure of minimal sets and we find a discontinuous Pitchfork bifurcation diagram. Then, we look at the global attractors, which exist, and once more we prove a discontinuous bifurcation phenomenon.

Joint work with T. Caraballo and J.A. Langa (Universidad de Sevilla) and A.M. Sanz (Universidad de Valladolid).

Jens STARKE, University of Rostock

Bifurcation analysis of collective behavior in particle models

The coarse behavior and its parameter dependence in complex systems is investigated. For this, a numerical multiscale approach called equation-free analysis is further improved in the framework of slow-fast dynamical systems. The method allows to perform numerical investigations of the macroscopic behavior of microscopically defined complex systems including continuation and bifurcation analysis on the coarse or macroscopic level where no explicit equations are available. This approach fills a gap in the analysis of complex real-world applications including particle models with intermediate number of particles where the microscopic system is too large for direct investigations of the full system and too small to justify large-particle limits. An implicit equation-free method is presented which reduces numerical errors of the analysis considerably. It can be shown in the framework of slow-fast dynamical systems, that the implicitly defined coarse-level time stepper converges to the true dynamics on the slow manifold. The method is demonstrated with applications to particle models of traffic as well as pedestrian flow situations. The results include an equation-free continuation of traveling wave solutions, identification of saddle-node and Hopf-bifurcations as well as two-parameter continuations of bifurcation points.

Dmitry TURAEV, Imperial College London

On the intersection of attractor and repeller.

In reversible systems and, more generally, systems with compact phase space, attractor and repeller may intersect, and this intersection can be robust. We discuss what this numerically discovered phenomenon could mean.

CONTRIBUTED TALKS

Peter ASHWIN, University of Exeter

Noise-induced tipping cascades

This talk will discuss some progress in understanding general principles of how coupling between dynamical systems that are on the threshold of tipping can lead to transitions in tipping behaviour from “independent tipping” to “domino effects” of various types, depending on coupling. We argue that the sequence of tipping is an emergent property of the coupled stochastic system.

Joint work with J. Creaser and K. Tsaneva-Atanasova (Exeter).

Peter GIESL, University of Sussex

Construction of a contraction metric by mesh-free collocation of a matrix-valued PDE

A Riemannian contraction metric can be used to prove existence and uniqueness of an equilibrium (or periodic orbit) of an autonomous ODE and determine a subset of its basin of attraction without requiring information about its position. Moreover, a contraction metric is robust to small perturbations of the system, which do not cause a bifurcation.

We will show how a contraction metric for an equilibrium can be characterised as a matrix-valued function, satisfying a certain linear PDE, and can then be approximated by mesh-free collocation. We use a recent extension of mesh-free collocation of scalarvalued functions, solving linear PDEs, to matrix-valued ones.

Partly work with Holger Wendland (Bayreuth, Germany).

Moussa NDOUR, TU Dresden

Exploring finite time qualitative changes of the autononomous and nonautonomous flow patterns using the Transfer operator: The double gyre model.

In this talk I am going to identify the early indicators of the finite time bifurcation of the area-preserving transitory double gyre model through transfer operator approach and, in perspectives, discuss the dynamic isoperimetry as a potential method to study more general nonautonomous systems.

Boumediene HAMZI, AlFaisal University

Kernel Methods and the Maximum Mean Discrepancy for Seizure Detection

We introduce a data-driven method for seizure detection drawing on recent progress in Machine Learning. The method is based on embedding probability measures in a high (or infinite) dimensional reproducing kernel Hilbert space (RKHS) where the Maximum Mean Discrepancy (MMD) is computed. The MMD is metric between probability measures that is computed as the difference between the means of probability measures after being embedded in an RKHS. Working in RKHS provides a convenient, general functional-analytical framework for theoretical understanding of data. We apply this approach to the problem of seizure detection.

Iacopo P. LONGO, Universidad de Valladolid

Strong and weak L^p_{loc}-type topologies for Carathéodory functions with applications in the study of non-autonomous ODEs

We consider spaces of Carathéodory functions and give conditions so that the skew product flow generated by Carathéodory non-autonomous ordinary differential equations (ODEs) is continuous. In particular, we will show optimal theorems for both strong and weak topologies of L^p_{loc}-type in such a setting.

As an application of such results, we show how suitable conditions on the solutions of an initial system $\dot{x} = f(t,x)$ allow to deduce the existence of a bounded pullback attractor for all the systems belonging to either the alpha limit set of f, the omega limit set of f, or the whole hull of f. Sufficient conditions for the existence of a pullback and a global atractor for the entire skew-product flow are also given. Such a formulation provides an interesting connection point between the dynamical theories of random and continuous skew-product semiflows with potential applications in Science and Engineering.

Joint work with Prof. Sylvia Novo and Prof. Rafael Obaya.

Kathrin PADBERG-GEHLE, Leuphana Universität Lüneburg

Probabilistic and discrete methods for the computational study of coherent behavior in flows

Transfer operator based numerical schemes have only recently been recognized as powerful tools for analyzing and quantifying transport processes in time-dependent flows. Central to this probabilistic concept are coherent sets, mobile regions in phase space that move about with minimal dispersion. Coherent sets can be efficiently identified via Perron-Frobenius operators (or transfer operators), which can be approximated within a set-oriented numerical framework.

While transfer operator based schemes require high resolution trajectory data, spatio-temporal clustering algorithms have been proven to be very effective for the extraction of coherent sets directly from sparse and possibly incomplete trajectory data. In particular, a discrete representation of the dynamics in terms of a trajectory network can be used to build a computationally very attractive and flexible approach.

In this contribution, we will give an introduction to the probabilistic transfer operator based concepts and the recently proposed discrete trajectory based approaches for the computational study of coherent behavior in flows. We will demonstrate the applicability of these methods in a number of example systems.

Ilya PAVLYUKEVICH, Friedrich Schiller University Jena

Small noise vibrations of a one-dimensional system with non-linear friction

We consider a second-order Langevin equation for the motion of a particle subject to a non-linear friction force being a power of the particle's velocity, $F = -|v|^\beta \ sign(v)$, $\beta \in \mathbb{R}$, and random vibrations. We determine the law of the displacement process in the limit of the small noise amplitude.

Joint work with Alexei Kulik (Kiev)

Lucía PÉREZ, Universidad de Oviedo

Homoclinic structure of the Hindmarh-Rose neuron model

The Hindmarsh-Rose model,

$$\dot{x} = y - ax^3 + bx^2 + I - z$$
$$\dot{y} = c - dx^2 - y$$
$$\dot{z} = \varepsilon\,(s(x - x_{rest}) - z)$$

is known to exhibit different behaviours generic for biological neurons. We study its dynamics considering a triparametric space (b, I, ε). Particular attention is paid to the changes occurred due to the system moving far from the singular case when $\varepsilon = 0$. We show how the structure of spike-adding still persists far from the slow-fast scenario but the geometry of the bifurcations changes notably. The shape of the homoclinic bifurcation curves and the existence of inclination-flip points are the main changes in the bifurcation diagrams for different values of ε. These transformations seem to be linked to the way in which the spike-adding process occurs, the evolution from fold/hom to fold/Hopf bursting behaviour and also with the way in which the chaotic regions evolve as the time scale of the slow variable increases.

Paul RITCHIE, Exeter University

How fast to turn around: preventing tipping after a system has crossed the tipping threshold

We investigate a scenario where a parameter of a dynamical system is changed such that it briefly crosses a tipping threshold but then returns to a value with stable equilibrium. We derive an approximation for the admissible distance from and time spent over the threshold before the system tips. We also give a result on the probability of tipping when the system is subject to additive noise. Our results are demonstrated for a two-dimensional model of the Indian Summer Monsoon (a simplification of a model by Zickfeld).

Pablo RODRÍGUEZ-SÁNCHEZ, Wageningen University

Stability landscapes for weakly non-gradient fields

Stability landscapes are useful tools for understanding dynamical systems. Usually, stability landscapes are identified with the physical concept of scalar potential. Unfortunately, the conditions for those potentials to exist are quite restrictive for higher dimensional systems. Here we present a numerical method for decomposing any field in two terms, one that has an associated potential (the gradient part), and another one that cannot be fit to it (the curl part). If the magnitude of the curl part is small compared to the gradient part, we can still make use of the concept of stability landscapes. The curl part to gradient part ratio can be used as a measure of the error introduced.

Anna VANSELOW, University of Oldenburg

Too fast to cope with - collapse of a predator-prey system

Environmental variations due to climate change or anthropogenic influences might give rise to unexpected responses of ecosystems, particularly their rapid decline. We study a paradigmatic predator-prey system, the Rosenzweig-MacArthur model, and demonstrate that a fast decline of nutrients can lead to a collapse of the ecosystem. We assume that nutrients decrease with a ramping parameter linear in time. As a result, the stable equilibrium also changes in time. When nutrients decrease too fast, the system is not able to track the moving stable equilibrium and the system will undergo a rate-induced transition. In addition, we use the method described in Wieczorek, [2011] to compute the threshold which separates initial conditions that undergo rate-induced transitions from those initial conditions which track the moving equilibrium for a given rate of change. Further we demonstrate that predator-prey systems which undergo rate-induced transitions are at high risk of extinction because they are determined by low population densities after the transitions. Because small populations often collapse when they are exposed to fast environmental variations and diseases, the analysis of rate-induced transitions is needed to prevent endangered populations.

Joint work with Sebastian Wieczorek and Ulrike Feudel

Sebastian WIECZOREK, University College Cork

Tipping in ecosystems: Points of no return

We investigate tipping phenomena using an example of a low-dimensional but non-autonomous ecosystem model [Scheffer et al. *Ecosystems* 11 2008], where parameter shifts describe slow and transient changes in plant growth or herbivore mortality over time. We define tipping as a critical transition from a herbivore-dominating stable state to a plant-only stable state, and study this transition in the tipping diagram of the magnitude vs. the rate of the parameter shift. The main focus is on the interaction between bifurcation-induced tipping (B-tipping) due to critical magnitudes of the parameter shift, and rate-induced tipping (R-tipping) due to critical rates of change of the parameter.

Firstly, we analyse monotone parameter shifts and obtain a tipping diagram that can be explained in terms of (i) the bifurcation diagram of the corresponding autonomous system where all parameters are fixed in time, and (ii) forward basin instability of the moving herbivore-dominating stable state. Next, we ask: *Given a monotone parameter shift that exceeds a critical magnitude or rate of change, can tipping be avoided by reversing the trend in the parameter change?* Specifically, we introduce the notion of a *point of no return* past which tipping can no longer be avoided upon the reversal in the trend of the time-varying parameter. The analysis of non-monotone parameter shifts uncovers an intriguing tipping diagram with multiple critical rates and non-trivial dependence of points of no return on the amplitude and rate of the parameter shift. We demonstrate that such tipping diagram is model-independent in the sense that it is characteristic of non-monotone parameter shifts across a generic steady bifurcation in the corresponding autonomous system.

Chun XIE, University College Cork

Rate-induced Critical Transitions

Critical transitions or tipping points are sudden and unexpected changes in the state of a complex system with time-varying external input, which are known to occur in climate, engineering, biology, social science and etc. Generally, there three categories of tipping, bifurcation-induced, noise-induced, and rate-induced(R-tipping). Mathematically, R-tipping is a genuine nonautonomous instability that cannot be explained by the classical bifurcation theory and requires an alternative approach.

At the first stage of the project, a time compactification method is developed for R-tipping analysis in nonautonomous system with asymptotically constant inputs. This method allows to study the critical rate of the change of the time-varying input in terms of a heteroclinic connection, which greatly simplifies the analysis. The methodology has been applied to canonical ODE examples of reversible and irreversible R-tipping, with threshold and quasi-threshold. In the next stage, the project will focus on the analysis of R-tipping in agent-based models of terrestrial and aquatic ecosystems.

POSTERS

Sajjad Bakrani Balani, Imperial College London

On the dynamics of super-homoclinic orbits

An orbit of a discrete (continuous) dynamical system which tends to a fixed point (equilibrium state) for both forward and backward time is called a homoclinic orbit. More precisely, a homoclinic orbit lies in the intersection of the stable and the unstable invariant manifold of the fixed point (equilibrium state). One of the main objects associated to homoclinic orbits which its presence can lead to rich dynamics is homoclinic to homoclinic or simply superhomoclinic orbit. Superhomoclinic orbits are those orbits which tend to a homoclinic orbit for both forward and backward time. More rigorously, they lie in the intersection of the stable and unstable invariant manifolds of a homoclinic orbit. It is a well-known result by Shilnikov and Smale that the existence of a transverse homoclinic orbit to a fixed point implies the existence of a horseshoe near that fixed point (the same is true for continuous systems). Despite the similarities between homoclinic and superhomoclinic orbits, it is not obvious if superhomoclinic orbits also possess horseshoes or not. We are trying to answer this question in a specific toy model. In this presentation, it will be explained how cone criterion and Shilnikov boundary value problem technique are applied on the problem to obtain some preliminary results.

Johannes LOHMANN, University of Copenhagen

Likelihood-free Bayesian model comparison of stochastic dynamical systems and application to paleoclimate data

Bayesian model comparison allows for a natural implementation of the law of parsimony, which guards against overfitting of models with large numbers of parameters to data. When using time series models given by stochastic differential equations, the likelihood function needed for Bayesian model comparison is not known. In order to proceed, one needs to use additional assumptions or approximations on the likelihood, or use a completely likelihood-free approach.

Motivated by our application to paleoclimate data, we avoid calibrating models by time series fitting, but rather focus on the most important qualitative features of the data, as captured by summary statistics. We thus employ a likelihood-free method that enables Bayesian model comparison using summary statistics, which is known as Approximate Bayesian Computation. Specifically, a combination of Approximate Bayesian Computation and sequential importance sampling is used to estimate posterior parameter distributions and Bayes' factors of competing models.

The method is applied to the NGRIP ice core record from Greenland, which displays the most pronounced variability in the climate during the last glacial period, namely the socalled Dansgaard-Oeschger (DO) events. Different stochastic dynamical systems are compared, which represent different dynamical mechanisms that might be underlying the abrupt climate changes observed in the ice core record.

Joint work with Peter Ditlevsen.

Holger METZLER, Max Planck Institute for Biogeochemistry, Jena

What is the entropy of a dissipative system?

The transport of biogeochemical elements (e.g., carbon, nitrogen, phosphorus) is often described by compartmental systems. Our main goal is to find useful metrics to compare the complexity of different compartmental systems. In the case of well-mixed compartments and donor-controlled, time-independent fluxes such systems are linear autonomous systems of ordinary differential equations of the type

$$\frac{d}{dt}\,\boldsymbol{x}(t) = \mathrm{B}\,\boldsymbol{x}(t) + \boldsymbol{u}\,, \qquad t > t_0\,, \qquad \boldsymbol{x}(t_0) = \boldsymbol{x_0} \in \mathbb{R}^d\,. \tag{1}$$

Owing to the law of conservation of mass, the matrix B is required to be compartmental:

(i) all diagonal entries are nonpositive;
(ii) all off-diagonal entries are nonnegative,
(iii) all column sums are nonpositive.

Furthermore, B is assumed to be nonsingular. Then the system contains no traps and all particles that enter the system will eventually leave it. Consequently, the compartmental system is dissipative and even globally asymptotically stable. It follows that both topological and metric entropy of all compartmental dynamical systems described by (1) are zero. As a result, these classical entropy notions cannot be used for complexity analysis.

The goal of our presentation is to introduce and discuss a reasonable notion of entropy of compartmental systems. We extend system (1) by one dimension and interpret the extended matrix $\widetilde{\mathrm{B}}$ as the generator of a continuous-time Markov chain. This Markov chain describes the journey of a single particle through the system from the particle's entry until its exit. This journey can be described by a path

$$Z = \big((C_1\,, T_1), (C_2\,, T_2)\,, \dots, (C_N\,, T_N)\big)\,,$$

where $(C_i)_i$ is the sequence of visited compartments and the T_i are the associated sojourn times. The Shannon information entropy of the random path Z can be used to define the entropy of the compartmental system and works as a measure of complexity. We then extend this approach to nonautonomous and nonlinear systems. In particular the extension to nonlinear systems gives raise to the hope to find a relation to the classical notions of topological and metric entropy of dynamical systems.

Joint work with Carlos A. Sierra.

Karl Nyman, University of Copenhagen

Aspects of the Middle Pleistocene Transition as due to frequency locking

Here, we aim to clarify, using a forced and ramped oscillator model introduced by (Crucifix 2012) the following questions:
Q1: How is additive forcing on either ice volume or a state variable expressed in an ice volume record?
Q2: How is synchronisation affected by choice of forced variable?

Damian SMUG, University of Exeter

Predicting financial market crashes using ghost singularities

We analyse the behaviour of a non-linear model of coupled stock and bond prices exhibiting periodically collapsing bubbles. By using the formalism of dynamical system theory, we explain what drives the bubbles and how foreshocks or aftershocks are generated. A dynamical phase space representation of that system coupled with standard multiplicative noise rationalises the log-periodic power law singularity pattern documented in many historical financial bubbles. The notion of `ghosts of finite-time singularities' is introduced and used to estimate the end of an evolving bubble, using finite-time singularities of an approximate normal form near the bifurcation point. We test the forecasting skill of this method on different stochastic price realisations and compare with Monte Carlo simulations of the full system. Remarkably, the former is significantly more precise and less biased. Moreover, the method of ghosts of singularities is less sensitive to the noise realisation, thus providing more robust forecasts.