a CRITICS Workshop on

DYNAMICAL SYSTEMS AND CRITICAL TRANSITIONS

University of Helsinki, 4 August, 2017

Program

8:30 - 9:00 **Registration**

Morning session

9:00 - 9:20 **Olli Hella** (University of Helsinki): 'Multivariate normal approximation for time-dependent dynamical systems'

9:20 - 9:40 **Chun Xie** (University College Cork): 'R-tipping with threshold and quasi-threshold'

9:40 - 10:00 **Juho Leppänen** (University of Helsinki): 'Functional correlation decay for intermittent maps'

10:00 - 10:30 **Maximilian Engel** (Imperial College London): 'Quasi-stationary dynamics and bifurcations of random dynamical systems'

Coffee Break (10:30 - 11:00)

11:00 - 11:25 **Iacopo P. Longo** (University of Valladolid): 'A brief survey on Carathéodory topological spaces and ODEs'

11:25 - 11:50 **Flavia Remo** (Friedrich Schiller University Jena): 'Early warning signals for bifurcations in skew product systems'

11:50 - 12:10 **Kalle Timperi** (Imperial College London): 'Numerical methods and uncertainty quantification for the Generalized Langevin Equation'

12:10 - 12:40 **Mikko Stenlund** (University of Helsinki): 'Quasistatic dynamical systems'

Lunch (12:40 - 14:00)

Afternoon Session

14:00 - 14:20 **Vadim Kulikov** (University of Helsinki): 'Are cognitive systems dynamical?'

14:20 - 14:40 **Damian Smug** (University of Exeter): 'A generalized 2D-dynamical mean-field Ising model with a rich set of bifurcations'

14:40 - 15:00 **Courtney Quinn** (University of Exeter): 'Identifying model sensitivity through finite-time Lyapunov exponents'

15:00 - 15:30 **Michael Hartl** (Imperial College London): 'Bifurcations in stochastic approximation dynamics'

Evening program

16:00 - 17:45 Sauna (Kotiharjun Sauna, Harjutorinkatu 1)

17:45 - 18:30 A walk in Kallio

18:30 - 21:00 Dinner at restaurant Cella (Fleminginkatu 15)

Abstracts:

Damian Smug: A generalized 2D-dynamical mean-field Ising model with a rich set of bifurcations

The Ising model and its mean-field version have a time-honoured history in economics, sociology and finance, since its introduction as a mathematical model of ferromagnetism in statistical mechanics in 1920. In finance, the spins represent agents who have several options and base their decisions on their own judgements, external news and social influences. We study an extended version of the dynamical mean-field equation of the Ising model in which the external field is endogenized to represent a smoothed moving average of the past state variable. This model stands for a simplification of the interplay between instantaneous social imitation and past trends in social coordinations. We show the existence of a rich set of bifurcations as a function of the two parameters quantifying the relative importance of instantaneous versus past social opinions on the formation of the next value of the state variable. Certain parameter regimes even exhibit chaotic behaviors.

Chun Xie: R-tipping with threshold and quasi-threshold

Critical transitions or tipping points are sudden and unexpected changes in the state of a complex system with time-varying inputs, generally in three categories, including bifurcation-induced, noise-induced, and rate-induced(R-tipping). This project is focused on R-tipping, which is a genuine nonautonomous instability that cannot explained by classical bifurcation theory and requires an alternative approach. In the first stage, a time compactification method is developed for R-tipping analysis with asymptotically constant inputs in terms of heteroclinic connection, so as to identify the critical rate of the change of time-varying inputs. And the methodology has been applied to canonical ODE examples. And the examples of R-tipping, with threshold defined by the manifolds of equilibria, and the quasi-threshold defined by the manifolds of the folded singularities would be introduced.

Maximilian Engel: Quasi-stationary dynamics and bifurcations of random dynamical systems

We look at Markov processes that induce a random dynamical system evolving in a domain with forbidden states constituting a trap. The process is said to be killed when it hits the trap and it is assumed that this happens almost surely. We investigate the behavior of the process before being killed, asking what happens when one conditions the process to survive for a long time. The topic goes back to the pioneering work by Yaglom in 1947 but in recent years new ideas have been developed. We discuss concepts like quasi-stationary and quasi-ergodic distributions, calling the associated random dynamics quasi-stationary or quasi-ergodic if such distributions exist. Given their existence, we can define average Lyapunov exponents and the Dichotomy spectrum of the random dynamical system with killing and describe the bifurcation behavior of typical examples of stochastic bifurcation theory within this environment. The underlying philosophy is to exhibit the local character of random bifurcations for stochastic differential equations which are usually hidden in the global analysis. We further relate these concepts to dynamical systems with holes.

Iacopo P. Longo: A brief survey on Carathéodory topological spaces and ODEs

In this joint work with Rafael Obaya and Sylvia Novo, new strong and weak topologies for Carathéodory functions are presented, and applications to non-autonomous ordinary differential equations are outlined. In particular, we give optimal conditions for building a continuous skew-product flow. Such a theory allows one to use the tool of topological dynamics to modelize and analyze phenomena that are regulated by unbounded or discontinuous vector fields with potential applications in control theory, biology and mechanics.

Kalle Timperi: Numerical methods and uncertainty quantification for the Generalized Langevin Equation

The generalized Langevin equation (GLE), a stochastic integro-differential equation, has recently gained popularity as a more realistic model for several phenomena in different contexts, including anomalous diffusion in biological fluids, microrheology, heat transport within nano-scale devices, and nuclear quantum effects. In climate modeling the GLE arises through the Mori-Zwanzig formalism as the equation describing the coarse-grained reduced order model of an initially high-order model. The advantage of the GLE over the conventional Langevin equation is that it allows for the incorporation of temporally non-local drag forces through an integration kernel in the diffusion term. This also poses new challenges for both the analytical and numerical treatment of the dynamics described by the equation. Also, the applicability of existing numerical schemes in practical contexts requires careful analysis of the measurement and modeling errors associated with these techniques. In this talk we provide an introduction to the GLE model and discuss potential directions of research.

Vadim Kulikov: Are cognitive systems dynamical?

20 years ago Deep Blue (DB) defeated Kasparov in chess utilizing extensive search strategies and rule and memory based symbolic artificial intelligence. The output of DB was a string telling which piece to move and where (e.g. Ke4). But if DB was required to move the pieces itself (with a robot hand etc..), it would have lost not only to Kasparov, but to any 5 year-old child who knows the rules of chess. Why did something so simple to humans (moving objects) turn out so difficult for computers and vice versa? The short answer is that our model of a cognitive system was wrong. In the past two to three decades both cognitive science and artificial intelligence have been rejecting ideas of rule-based symbol manipulation as a basis for cognition and moving towards more dynamic models of cognitive processes where action and perception are unified into a dynamic coupling of sensorimotor systems with the environment. The conjecture is that dynamic approach is useful not only in understanding our physical behaviour, but indeed also in understanding abstract thinking. Drawing on recent developments in philosophy of mind, cognitive science and artificial intelligence, I will ask the following question in this presentation: Can dynamical systems help us to create a (unifying) mathematical theory of cognition?