Short course on slow-fast systems

Christian Kühn, Technische Universität München

The course covers the basic theory of fast-slow dynamical systems from a geometric, numerical and stochastic perspective. We start with the geometric theory and introduce critical/slow manifolds as well as Fenichel's Theorem for normally hyperbolic invariant manifolds. Then we are going to introduce singularities, where normal hyperbolicity is lost. At these points the fast and slow scales mix. Then we consider briefly the transition between fast and slow motions in the case of singularities. Next, we cover oscillations in fast-slow systems arising in many applications. In this context, a decomposition of the dynamics leads us to understand different mechanisms to obtain MMOs and bursting. Then we proceed to numerical aspects and continuation methods to attack larger models. In the last part, we cover stochastic aspects and early-warning signs for tipping points in stochastic fast-slow systems. In summary, we are going to acquire tools, which are necessary to analyze many classes of applied problems.

TASK: Each participant has to find/prepare one multiple time scale system close to her/his interests *in advance*. The system should be ODEs, preferably of very small dimension d between 2 and 4. You have to write down the system, note down the main choice of parameters, and have an ODE integrator ready for the system (e.g. implement it in MatLab with ode15s).

In the practical part of the course, you are going to apply the methods from the first part to your own system.