Short course on equation-free approaches in complex systems

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Complex systems, i.e. systems with many degrees of freedom, often show a low-dimensional macroscopic behavior which is of interest to be analyzed. Of particular interest is a macroscopic bifurcation analysis, i.e. the analysis of the qualitative macroscopic behavior depending on relevant parameters. Often there are no explicit equations available on the macroscopic level but a microscopic model of the complex system can be accessed numerically. The so-called equation-free analysis or coarse analysis allows to perform the analysis of the macroscopic behaviour without explicitly given equations on that level by suitably chosen short simulation bursts of the microscopic model. This approach fills a gap between the methods for a detailed analysis of models with few degrees of freedom and pure simulations of complex real-world applications with many degrees of freedom. An implicit equation-free method is presented which reduces numerical errors of the analysis considerably. It can be shown in the framework of slow-fast dynamical systems, that the implicitly defined coarse-level time stepper converges to the true dynamics on the slow manifold. The method is demonstrated for applications to particle models of traffic as well as pedestrian flow situations. The results include an equation-free continuation of traveling wave solutions, identification of saddle-node and Hopf-bifurcations as well as two-parameter continuations of bifurcation points.

The lectures will include

1) some basics of numerical bifurcation analysis, in particular continuation techniques

2) hands-on exercises with MATLAB to perform a pseudo-arclength continuation of simple low-dimensional dynamical systems

3) the mathematical background of the equation-free approach

4) hands-on exercises with MATLAB to perform an equation-free analysis on a selected microscopic model