Critical Transitions in Complex Systems: Mathematical theory and applications.

Winter school program
5-9 March 2018

Time	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
09:00-10:30	Padberg: remarks Kühn (09:30)	Starke	Starke	Padberg-Gehle	Padberg-Gehle
10:30-11:00	**Coffee break**				
11:00-12:30	Kühn	Starke	Starke	Padberg-Gehle	Padberg-Gehle
12:30-14:30	**Lunch break**				
14:30-15:30	Kühn	Starke	Starke	Padberg-Gehle	Discussion\ Ex
15:30-16:00	**Coffee break**				
16:00-17:30	Discussion\ Ex	Discussion\ Ex	Discussion\ Ex	Discussion\ Ex	Discussion\ Ex
18:30-20:00	**Dinner** [CRITICS Students Meeting (on Wednesday)]				

Topics for the School

Short course on slow-fast systems
Christian Kühn, (Technische Universität München)

The course covers the basic theory of fast-slow dynamical systems from a geometric, numerical and stochastic perspective. We start with the geometric theory and introduce critical/slow manifolds as well as Fenichel's Theorem for normally hyperbolic invariant manifolds. Then we are going to introduce singularities, where normal hyperbolicity is lost. At these points the fast and slow scales mix. Then we consider briefly the transition between fast and slow motions in the case of singularities. Next, we cover oscillations in fast-slow systems arising in many applications. In this context, a decomposition of the dynamics leads us to understand different mechanisms to obtain MMOs and bursting. Then we proceed to numerical aspects and continuation methods to attack larger models. In the last part, we cover stochastic aspects and early-warning signs for tipping points in stochastic fast-slow systems. In summary, we are going to acquire tools, which are necessary to analyze many classes of applied problems.

TASK: Each participant has to find/prepare one multiple time scale system close to her/his interests *in advance*. The system should be ODEs, preferably of very small dimension d between 2 and 4. You have to write down the system, note down the main choice of parameters, and have an ODE integrator ready for the system (e.g. implement it in MatLab with ode15s).

In the practical part of the course, you are going to apply the methods from the first part to your own system.

Short course on equation-free approaches in complex systems
Jens Starke, (Universität Rostock)

Complex systems, i.e. systems with many degrees of freedom, often show a low-dimensional macroscopic behavior which is of interest to be analyzed. Of particular interest is a macroscopic bifurcation analysis, i.e. the analysis of the qualitative macroscopic behavior depending on relevant parameters. Often there are no explicit equations available on the macroscopic level but a microscopic model of the complex system can be accessed numerically.

The so-called equation-free analysis or coarse analysis allows to perform the analysis of the macroscopic behaviour without explicitly given equations on that level by suitably chosen short simulation bursts of the microscopic model. This approach fills a gap between the methods for a detailed analysis of models with few degrees of freedom and pure simulations of complex real-world applications with many degrees of freedom. An implicit equation-free method is presented which reduces numerical errors of the analysis considerably. It can be shown in the framework of slow-fast dynamical systems, that the implicitly defined coarse-level time stepper converges to the true dynamics on the slow manifold. The method is demonstrated for applications to particle models of traffic as well as pedestrian flow situations. The results include an equation-free continuation of traveling wave solutions, identification of saddle-node and Hopf-bifurcations as well as twoparameter continuations of bifurcation points.

The lectures will include:

1. Some basics of numerical bifurcation analysis, in particular continuation techniques.
2. Hands-on exercises with MATLAB to perform a pseudo-arclength continuation of simple low-dimensional dynamical systems.
3. The mathematical background of the equation-free approach.
4. Hands-on exercises with MATLAB to perform an equation-free analysis on a selected microscopic model.

Short course on numerical methods for stochastic processes
Kathrin Padberg-Gehle, (Leuphana Universität Lüneburg)

A stochastic process is a collection of random variables parameterized by time. It often serves as a mathematical description of a system or phenomenon that is influenced by some randomness. Examples include the growth of a bacterial population, the motion of a molecule, or the dynamics of the financial market.
Often stochastic processes are described by stochastic differential equations (SDEs). The mathematical treatment of SDEs requires a specific calculus and numerical simulation methods for SDEs have to be consistent with the corresponding stochastic calculus.

This course serves as an introduction to the theory and especially the numerical analysis of stochastic processes.

Topics:

- Generation of random numbers and variables
- Monte Carlo simulation
- Variance-reduction techniques
- Continuous time stochastic processes
- Stochastic differential equations
- Numerical solution of SDEs
- $\cdots$

In the practical part of the course, we will apply the results from the lectures to different example systems. This also means that we will implement and run the models and algorithms in MATLAB.

References

1. Kloeden/Platen: Numerical solutions of stochastic differential equations (Springer)
2. Asmussen/Glynn: Stochastic simulation: algorithms and analysis (Springer)
3. Øksendal: Stochastic differential equations - an introduction with applications (Springer)