Critical Transitions in Complex Systems: Mathematical theory and applications.

Conference program
12-16 March 2018

Monday 12 March	**Speaker**	**Title**
09:00-10:00	Junge	Robust FEM-based extraction of finite-time coherent sets using scattered, sparse, and incomplete trajectories
10:00-10:45	Coffee break	Coffee break
10:45-11:45	Bahsoun	Linear response for random compositions of maps
11:45-12:15	Bittracher	Data-driven Coarse Graining of Complex Molecular Systems: The Transition Manifold Approach
12:15-14:30	Lunch	Lunch
14:30-15:30	Pavylukevich	Probabilistic methods for Levy-driven dynamics
15:30-16:15	Coffee break	Coffee break
16:15-16:45	Fuhrmann	Introduction to Amorphic complexity
16:45-17:15	Herpich	Collective power: Minimal model for thermodynamics of nonequilibrium phase transitions
17:15-18:30	Individual discussions	Individual discussions
18:30-20:00	Dinner	Dinner

Tuesday 13 March	**Speaker**	**Title**
09:00-10:00	Von der Heydt	The Eocene-Oligocene transition, observations, mechanisms, cascading tipping?
10:00-10:45	Coffee break	Coffee break
10:45-11:15	Quinn	Exploration of bistability in a paleoclimate delay model
11:15-12:15	Sieber	Mid-pleistocene transition as a collision between non-autonomous attractor and saddle
12:15-14:30	Lunch	Lunch
14:30-15:30	Sierra	Understanding the global carbon cycle using compartmental systems: ages and transit time distributions
15:30-16:15	Coffee break	Coffee break
16:15-16:45	Longo	Study of staibility of Carathéodory compartmental systems
16:45-18:30	Excursion	Brewery
18:30-20:00	Dinner	Dinner
20:00-21:00	**CRITICS** meeting	Career/funding opportunities

Wednesday 14 March		
09:00-10:00	Dellnitz	Glimpse of the Infinite- the Approximation of Invariant Sets for Delay and Partial Differential Equations
10:00-10:45	Coffee break	Coffee break
10:45-11:45	Ditlevsen	Glacial cycles as complex climate responses to astronomical forcing.
11:45-12:15	Hamzi	Kernel Methods and the Maximum Mean Discrepancy for some Slow-Fast Systems with Critical Transitions
13:00-14:30	Lunch	Lunch
14:30-15:00	Peitz	Controlling the Navier-Stokes equations using low-dimensional bilinear approximations obtained from data
15:00-15:30	Kaiser	Sparse regression for modeling and control of nonlinear dynamical systems
15:30-16:15	Coffee break	Coffee break
16:15-16:45	Hartl	Bifurcations in one dimensional stochastic approximations
16:45-17:15	Lohmann	Cascading tipping points in paleoclimate
17:15-18:30	Individual discussions	Individual discussions
18:30-20:00	Dinner	Dinner
20:00-21:00	**CRITICS** Steering Committee	Meeting

Thursday 15 March	**Speaker**	**Title**
09:00-10:00	Dakos	The Inevitability of Surprise under Global Environmental Change
10:00-10:45	Coffee break	Coffee break
10:45-11:45	Van Nes	Yes, When can we identify causal feedbacks?
11:45-12:15	Meyer	Why doesn't biodiversity recover in formerly polluted grasslands?
12:15-12:45	Xie	R-tipping with quasi-threshold in slow-fast system
12:45-14:30	Lunch	Lunch
14:30-18:30	Excursion	Goslar
18:30-20:00	Dinner	Dinner

Friday 16 March	**Speaker**	**Title**
09:00-10:00	Hartmann	Adaptive importance sampling of rare events
10:00-10:45	Coffee break	Coffee break
10:45-11:45	Rasmussen	Critical transitions in set-valued dynamical systems
11:45-12:15	Falkena	Verification of Discrete Delay Models for ENSO using Projection Methods
12:15-14:30	Lunch	Lunch
14:30-18:30	Individual discussions	Individual discussions
18:30-20:00	Dinner	Dinner

Abstracts of the talks

Robust FEM-based extraction of finite-time coherent sets using scattered, sparse, and incomplete trajectories

Oliver Junge (Technische Universität Mnchen)

Transport and mixing properties of aperiodic flows are crucial to a dynamical analysis of the flow, and often have to be carried out with limited information. Finite-time coherent sets are regions of the flow that minimally mix with the remainder of the flow domain over the finite period of time considered. In the purely advective setting this is equivalent to identifying sets whose boundary interfaces remain small throughout their finite-time evolution. Finite-time coherent sets thus provide a skeleton of distinct regions around which more turbulent flow occurs. They manifest in geophysical systems in the forms of e.g. ocean eddies, ocean gyres, and atmospheric vortices. In real-world settings, often observational data is scattered and sparse, which makes the difficult problem of coherent set identification and tracking even more challenging. We develop three FEM-based numerical methods to efficiently approximate the dynamic Laplace operator, and introduce a new dynamic isoperimetric problem using Dirichlet boundary conditions. Using these FEM-based methods we rapidly and reliably extract finite-time coherent sets from models or scattered, possibly sparse, and possibly incomplete observed data.

Linear response for random compositions of maps

Wael Bahsoun (Loughborough University, London)

In this talk I will give a brief overview of progress on the topic of linear response. For systems whose transfer operators admit a uniform spectral gap on a suitable Banach space, it is well known that under reasonable conditions on the transfer operator and the noise, the underlying system admits linear response. Recently, there has been progress on linear response for systems whose transfer operators do not admit a spectral gap. I will discuss this topic in details, both in the deterministic and random settings. This last part is a joint work with Marks Ruziboev and Benoit Saussol.

Data-driven Coarse Graining of Complex Molecular Systems: The Transition Manifold Approach

Andreas Bittracher (Freie Universität Berlin)

For many complex molecular systems, one is interested primarily in the behavior on long time scales compared to the fundamental laws that govern the dynamics. For example, the forming of secondary and tertiary motifs in peptides and proteins happens on time scales that are 8-15 orders of magnitude larger than those of the basic atomic interactions. As these fast interactions however have to be resolved by classical numerical integrators, the direct simulation of the longtime dynamics remains out of reach for the foreseeable future. In these situations, efficient coarse graining schemes become necessary that systematically replace the original system by a surrogate dynamics on a reduced state space. The construction typically starts with the identification of good reaction coordinates, nonlinear observables of the full state space onto which the full system can be projected while preserving the long time scales. We have recently shown that, based on a specific reducibility property, the existence of good low-dimensional reaction coordinates is guaranteed. This property postulates the existence of a so-called transition manifold, a low-dimensional manifold in a specific function space, which can be seen as the backbone of the long time dynamics. Based on this theoretical framework, multiple numerical approaches for the computation of reaction coordinates have been developed, including a mesh-free, data-driven method applicable to commonly-available molecular trajectory data.

In this talk, both the theoretical concepts of the transition manifold, as well as the data-driven algorithm will be presented and demonstrated on a small yet realistic peptide system.

Probabilistic methods for Levy-driven dynamics

Ilya Pavylukevich (Friedrich Schiller University, Jena)

We will discuss how stochastic systems driven by Levy processes can be studied with the help of probabilistic methods. We will address such problems as first exit dynamics of systems driven by Levy flights, metastability, Levy-driven ratchets, and the famous Ito- Stratonovich dilemma which arises when one examines a dynamical system with a multiplicative noise.

Introduction to Amorphic complexity (Joint work with Maik Gröger and Tobias Jäger)

Gabriel Fuhrmann (Friedrich Schiller University, Jena)

This talk deals with the amorphic complexity, a recently introduced topological invariant that measures the complexity of dynamical systems in the regime of zero entropy. Its main purpose is to detect the very onset of disorder in the asymptotic behaviour. For instance, it gives positive value to Denjoy examples on the circle and Sturmian subshifts, while it is still zero for all isometries and MorseSmale systems.
If time permits, after discussing basic properties and examples, we may take a closer look at symbolic systems, where the amorphic complexity equals the box dimension of its maximal equicontinuous factor equipped with a naturally associated metric.

Collective power: Minimal model for thermodynamics of nonequilibrium phase transitions.

T.Herpich(1) J. Thingna,(1), and M. Esposito(1)

Tim Herpich (University of Luxembourg)

We establish a direct connection between the linear stochastic dynamics, the nonlinear mean-field dynamics, and the thermodynamic description of a minimal model of driven and interacting discrete oscillators [1,2]. This system exhibits at the mean-field level two bifurcations separating three dynamical phases: a single stable fixed point, a stable limit cycle indicative of synchronization, and multiple stable fixed points. These complex emergent behaviors are understood at the level of the underlying linear Markovian dynamics in terms of metastability, i.e. the appearance of gaps in the upper real part of the spectrum of the Markov generator [3]. Thermodynamically, the dissipated work of the stochastic dynamics exhibits signatures of nonequilibrium phase transitions over long metastable times which disappear in the infinite-time limit. Remarkably, it is reduced by the attractive interactions between the oscillators. When operating as a work-to-work converter, we find that the maximum power output is achieved far-from-equilibrium in the synchronization regime and that the efficiency at maximum power is surprisingly close to the universal linear regime prediction [4,5]. Our work builds bridges between thermodynamics of nonequilibrium phase transitions and bifurcation theory.

References:

[1] Herpich, T., Juzar, T., Esposito, M., 2018. arXiv:1802.00461
[2] Wood, K., et al., 2006. Phys. Rev. Lett., 96, 145701.
[3] Macieszczak, K., et al., 2016. Phys. Rev. Lett., 116, 240404
[4] Van den Broeck, C., 2005. Phys. Rev. Lett., 95, 190602.
[5] Esposito, M., et al., 2009. Phys. Rev. Lett., 102, 130602

The Eocene-Oligocene transition, observations, mechanisms, cascading tipping?

Anna S. von der Heydt (Utrecht University)

Over the last 65 million years, the Earths climate has undergone a large transition from a warm and ice-free greenhouse climate to an icehouse cli- mate with extensive ice sheets on both hemispheres. The gradual cooling may be seen as response to the overall slowly decreasing atmospheric CO_2- concentration due to weathering processes in the Earth System, however, continental geometry has changed considerably over this period and the long- term gradual trend was interrupted, by several rapid transitions and periods where temperature and greenhouse gas concentrations seem to be decoupled. The Eocene-Oligocene transition (34 Ma) reflects a first major phase of Antarctic ice sheet build-up and global climate cooling. In detail, the transition consists of two distinct steps in the oxygen isotope record, where the first reflects mostly ocean cooling, while during the second a large ice sheet has built up. Here we consider the possibility of two coupled critical transitions in explaining the two-step nature of the Eocene-Oligocene climate change. In this view, a leading bistable system undergoes a transition thereby changing the background conditions for a second following bistable system, which under the altered background conditions undergoes a critical transition. For the Eocene-Oligocene transition, the leading system could be a bistable ocean meridional overturning circulation, while the second following system reflects the bistable land-ice system coupled by the atmospheric CO_2 concentration. We investigate the possibility of cascading systems in a simple conceptual climate model. Moreover, the necessary condition of the existence of bistability in the ocean circulation is tested in an ocean general circulation model under Eocene continental boundary conditions.

Exploration of bistability in a paleoclimate delay model

Courtney Quin (University of Exeter)

We present a follow-up study of a delay differential equation (DDE) model for the Mid-Pleistocene Transition (having occurred between 1200 and 800 kyr BP). The model has a bistable region consisting of a stable equilibrium along with a large amplitude stable periodic orbit. When subjected to the quasiperiodic astronomi-cal forcing, the unforced stable states correspond to a small- and large-amplitude response. A parameter study shows the model exhibits a transition between these responses both in time and in forcing strength. To further understand the transition related to forcing strength, we investigate the behaviour of the model with periodic forcing. The quasiperiodic forced model displays a threshold behaviour when the forcing amplitude is increased - moving the model from a non-transitioning regime to a transitioning regime. Similar threshold behaviour is found when the periodic forc- ing amplitude is increased. We show that the forcing amplitude at which we observe the threshold is far away from any bifurcations. The only alternative mechanism is that the initial condition of our simulation moves from the basin of attraction of the small-amplitude response to the basin of the large-amplitude response. The threshold is also sensitive to the phase of the periodic forcing. (joint work with Jan Sieber and Anna S. von der Heydt)

Mid-pleistocene transition as a collision between non-autonomous attractor and saddle

Jan Sieber (University of Exeter)

This talk will try to make a connection between the observed sudden onset of large scale oscillations in a non-periodically forced model for ice ages (as reported in Courtney Quinn's talk) and work on saddle-node bifurcations (smooth or non-smooth) in quasi-periodically forced maps. We observe numerically that the non-autonomous perturbation of the autonomous saddle equilibrium collides with the non-autonomous perturbation of the autonomous stable equilibrium in our model (a delay-differential equation).

Understanding the global carbon cycle using compartmental systems: ages and transit time distributions

Carlos Sierra (Max Planck Institute for Biogeochemistry)

Studying the global carbon cycle is crucial to understand how human activities modify the climate system, in particular how the burning of fossil fuels may lead to bifurcations in the way carbon is exchanged among the atmosphere, the oceans, and the terrestrial biosphere. The global carbon cycle is simulated using complex computer models that track how carbon moves across different Earth system reservoirs. Only until recently, it was realized that carbon cycle models can be represented as compartmental dynamical systems, offering new opportunities to study carbon-climate interactions using concepts from dynamical system theory. In this contribution, I will show how we can generalize carbon cycle models as compartmental systems, and propose a classification of different models according to linearity and autonomy. The concepts of age and transit time of carbon across compartmental systems are introduced as useful diagnostics of model behavior. A correspondence between linear autonomous compartmental systems and absorbing continuous-time Markov chains is introduced and used to derive density distributions for ages and transit times. For non-linear non-autonomous compartmental systems, a semi-explicit method is introduced to derive age and transit time distributions for specific solution trajectories. Outstanding questions about stability and controlability of compartmental systems will be posed at the end of the presentation

Study of stability of Carathory compartmental systems.

Iacopo Paolo Longo (Universidad de Valladolid)

Due to their weak assumptions on regularity, non-autonomous ordinary differential equations (ODEs) and functional differential equations (FDEs) of Carathory type can be succesfully applied to model a wide range of real phenomena. Particularly, the talk will show how to model non-autonomous, non-linear compartmental systems and infer stability results of the associated mean age system. Additionally, we outline the abstract version of these results, giving sufficient conditions to prove the existence of a pullback attractor for any Carathory system with suitable features. Finally we also show how to propagate such pullback attractor if a continuous skew product semiflow can be defined.

Glimpse of the Infinite-the Approximation of Invariant Sets for Delay and Partial Differential Equations.

Michael Dellnitz (Universität Paderborn)

In this talk we present a novel numerical framework for the computation of finite dimensional invariant sets for infinite dimensional dynamical systems. With this framework we extend classical set oriented numerical schemes (for the computation of such objects in finite dimensions) to the infinite dimensional context. The underlying idea is to utilize appropriate embedding techniques for the reconstruction of invariant sets in a certain finite dimensional space. Finally, we illustrate our approach by the computation of attractors both for delay and for partial differential equations.

Glacial cycles as complex climate responses to astronomical forcing

Peter Ditlevsen (University of Copenhagen)

Through the past few million years large ice sheets have repeatedly grown and disappeared on the Northern hemisphere. These are the Pleistocene glaciations. They are related to the changing solar heating of the Earth due to changes in Earth's orbit and axis of rotation. The climate response to these changes is highly non-trivial and non-linear, expressing the complex nature of the climate system. Many aspects of glacial cycles still need a convincing explanation, one of the mysteries being the change from approximately 40 kyr (kilo year) glacial cycles to approximately 100 kyr cycles around 1 million years ago. This transition is called the middle Pleistocene transition (MPT). Here I present conceptual models to explain the dynamics of glacial cycles and possible dynamical causes of the MPT. I shall especially focus on a recently presented model, relating the MPT to a transcritical bifurcation in the structure of a generic critical/slow manifold for a fast-slow dynamical system.

Kernel Methods and the Maximum Mean Discrepancy for some Slow-Fast Systems with Critical Transitions.

Boumediene Hamzi (AlFaisal University)

We introduce a data-driven method for some systems with critical transitions drawing on recent progress in Machine Learning. The method is based on embedding probability measures in a high (or infinite) dimensional reproducing kernel Hilbert space (RKHS) where the Maximum Mean Discrepancy (MMD) is computed. The MMD is a metric between probability measures that is computed as the difference between the means of probability measures after being embedded in an RKHS. Working in RKHSs provides a convenient, general functional-analytical framework for theoretical understanding of data. We apply this approach to the problem of detecting critical transitions in some slow-fast systems. This is joint work with Sameh Mohamed (SUTD, Singapore) and Christian Kuehn (Technical University of Munich, Munich/Germany).

Controlling the Navier-Stokes equations using low-dimensional bilinear approximations obtained from data

Sebastian Peitz (Paderborn University)

In this talk we present a data based reduced order modeling approach for control of nonlinear PDEs which relies on the Koopman operator. We show that if the control enters linearly into the system, we can construct a bilinear surrogate model via linear interpolation between two Koopman operators corresponding to constant controls. Using a recent convergence result for Extended Dynamic Mode Decomposition, convergence of the reduced order model based control problem towards the true optimum can be guaranteed. The resulting feedback controller is used to control the two-dimensional flow around a cylinder in real-time.

Sparse regression for modeling and control of nonlinear dynamical systems

Eurika Kaiser (University of Washington)

High-dimensional, nonlinear, multi-scale phenomena, such as turbulence or the spread of infectious diseases, are ubiquitous; however, we still lack a good understanding of these as analytically tractable models remain an exception. The lack of simple equations and unprecedented amount of available high-fidelity data are leading to a paradigm shift in how we interact with complex nonlinear systems. Leading approaches stem from data-driven methods which have the potential to discover new mechanisms, models and control laws and are driven by the tremendous advances in computing power, new sensors and infrastructures, and advanced algorithms in machine learning. In this talk, I will discuss recent advances in data-driven, equation-free architectures leveraging sparsity-promoting techniques for the modeling and control of dynamical systems. One direction is connected to Koopman operator theory, which has emerged as a principled framework to obtain linear embeddings of nonlinear dynamics, enabling the estimation, prediction and control of strongly nonlinear systems using standard linear techniques. A data-driven architecture is presented for the identification of Koopman eigenfunctions using sparse regression and polynomial expansions, based on the partial differential equation governing the infinitesimal generator of the Koopman operator. It is shown that lightly damped eigenfunctions may be faithfully extracted using sparse regression; and how these intrinsic coordinates can be used to manipulate nonlinear systems using linear control theory.

Bifurcations in one dimensional stochastic approximations

Michael Hartl (Imperial College London)

Stochastic Approximations are a particular type of recursively defined stochastic processes. They emerged in the 50s, in works by RobbinsMonro and KieferWolfowitz, as a method of cancelling out measurement errors in repeated measurements. Since then, many more applications of stochastic approximations were found, e.g. they can be used to describe generalised Polya urn models or certain learning algorithms. They are characterized by a mean vector field, a sequence of step sizes and a sequence of noise variables. In this talk, I will show the influence of the latter of those ingredients on the convergence behaviour of the stochastic approximation. In particular, I will assume a uniform bound for the noise and will show how increasing this bound leads to a decreased stability of the possible limit points of the process.

Cascading tipping points in paleoclimate (joint work with D. Castellana)

Johannes Lohmann (University of Copenhagen)

Networks of dynamical systems, which when isolated allow for multi-stability, may exhibit cascades where a critical transition in one sub-system via noise-induced, rate-induced or bifurcation tipping triggers a critical transition in one or more other sub-systems. Real-world analogues might be present in the climate system, where several interacting sub-system have been proposed to potentially undergo chains of critical transitions under global warming scenarios.

One could furthermore speculate that certain abrupt climate changes in the past occurred as cascades of tipping points. To this end, we draw inspiration from recent general circulation model simulations of abrupt climate changes reminiscent of Dansgaard-Oeschger events and also from evidence gathered in ice core records and other paleoclimate proxies. Both data and models indicate that abrupt glacial climate change arises from the interaction of the atmospheric circulation pattern, sea ice and the Atlantic thermohaline circulation, and potentially due to a cascade of critical transitions or bifurcations occurring

in these systems.
Based on these ideas, we construct a coupled conceptual model consisting of a sea ice and thermohaline circulation components, both exhibiting bi-stability. Changing atmospheric conditions are modeled by either considering a drift in a bifurcation parameter or by additive noise. We employ this system as toy model to learn about cascading tipping points in general, as well as in relation to abrupt paleoclimate changes.
The sea ice model has a double fold bifurcation structure and is coupled in a feed-forward manner to the ocean model with a double fold that permits rate-induced tipping. In this on-going research, we describe the different tipping cascades that are possible in this model and assess the performance of early-warning signals due to critical slowing down and changes in cross-correlation in between the sub-systems.

The Inevitability of Surprise under Global Environmental Change

Vasilis Dakos (ETH Zurich)

Evidence is increasing that large scale abrupt changes in ecosystems, fisheries, oceanic circulation patterns, or even human physiology are examples of catastrophic transitions between different dynamical states typically referred to as tipping points. Recent theoretical findings suggest that distinct properties tend to rule system dynamics prior to such transitions. When quantified, these properties may be even used as indicators of loss of resilience for these systems. In this talk, I will present my work on how we can quantify resilience and detect tipping points in ecological systems. I will use mutualistic communities as an example of measuring resilience to stress in ecological networks. Lastly, I will outline some ideas on how we can expand such approaches to better understand tipping point responses in a changing but adaptive world.

Yes, When can we identify causal feedbacks?

Egbert van Nes (Wageningen University)

In this talk I will introduce a recent method for identifying causality in non-linear systems. This method is called Convergent Cross Mapping (CCM) (Sugihara et al. 2012) and it claims to be able to find bidirectional causality. I applied this method to ice-core data to find evidence for a causal feedback in the climate. However, these data turned out to be too strongly correlated to be decisive about bidirectional causality.

Why doesn't biodiversity recover in formerly polluted grasslands?

Katherine Meyer (University of Minnesota)

Earth's ecosystems take up excess nutrients from human sources ranging from fertilizer runoff to combustion by-products, and this nutrient loading can significantly alter ecosystem structure and function. Grassland experiments initiated in 1982 demonstrated that a decade of elevated nitrogen addition can cause biodiversity loss and exotic species invasion. Furthermore, biodiversity has failed to recover more than 20 years after cessation of high rates of nitrogen supply, suggesting the ecosystem has shifted to a new regime. Mathematical models can complement ongoing field experiments to investigate the mechanisms that reinforce the low-biodiversity community. Here I present a nonlinear ODE model that highlights the potential role of plant litter in preventing biodiversity recovery. Understanding why and how grasslands get stuck in a low-diversity state could help identify management levers that restore biodiversity not just in experiments but also in ecosystems worldwide that are recovering after pollution reduction.

R-tipping with quasi-threshold in slow-fast system

Chun Xie (University College Cork)

Rate-induced critical transition or rate-induced tipping(R-tipping) is a genuine nonautonomous instability that cannot be explained by the classical bifurcation theory. When R-tipping happens, the system fails to follow a moving stable state and undergoes a critical transition but only if the external conditions vary too fast. Here presents the analysis of R-tipping on a canonical slow-fast system. Through the developed compactification method for asymptotically autonomous system, and desingularization, a rigorous testable criteria for R-tipping on slow-fast system is given, and the classification of R-tipping with quasi-threshold is defined.

Adaptive importance sampling of rare events

Carsten Hartmann (Freie Universität Berlin)

I will introduce a numerical method for computing the optimal change of measure for certain classes of rare event simulation problems that appear in statistical mechanics, e.g. in molecular dynamics. The method is based on a representation of the rare event sampling problem as an equivalent (or: dual) stochastic optimal control problem, whose value function characterizes the optimal (i.e. minimum variance) change of measure. The specific duality behind the problem is then used to devise numerical algorithms for computing the optimal change of measure. In this talk I will describe two approaches in some detail that are built on a semi-parametric representation of the value function: a cross-entropy based stochastic approximation algorithm and a Monte-Carlo based least-squares discretisation of a related forward-backward stochastic differential equation. I will discuss the general approach, with a particular focus on the choice of the ansatz functions and the solution of high-dimensional problems, and illustrate the numerical method with simple toy examples.

Critical transitions in set-valued dynamical systems

Martin Rasmussen (Imperial College London)

We consider critical transitions in dynamical systems perturbed by bounded noise. Dynamical systems perturbed by bounded noise can be modeled either as random dynamical systems (describing the dynamics for each noise realisation) or set-valued dynamical systems (describing only the collection of all possible behaviour). We focus in this talk on set-valued dynamical systems, but similarities and differences to bifurcations in random dynamical systems will be highlighted. We demonstrate that under reasonable assumptions, there are only two different types of critical transitions in set-valued dynamical systems, and we characterise these critical transitions by means of collisions of attractor-repeller pair. Analogous questions are studied in the nonautonomous case.

Joint work with Jochen Bröcker, Giulia Carigi, Thai Son Doan, Christian Kuehn, Jeroen Lamb, Giuseppe Malavolta, Julian Newman, Christian Rodrigues.

Verification of Discrete Delay Models for ENSO using Projection Methods

Swinda Falkena (Universiteit Utrecht)

The El Niño Southern Oscillation (ENSO) is a well studied climate phenomenon occurring in the Pacific Ocean. Both models and data have been used to explain the variations in sea surface temperature. Suarez and Schopf (1988) proposed a simple discrete delay model for the phenomenon based on integration along wave characteristics. In this model a cubic nonlinearity is assumed, but no physical arguments for this form are presented. This model is able to produce a stable oscillation with a period slightly smaller than that of ENSO. Using the Mori-Zwanzig projection formalism a discrete delay model for ENSO can be derived starting from a simple two-strip model of the equatorial ocean. Assuming a nonlinearity in the original PDEs, the resulting discrete delay equation contains the nonlinearity assumed by Suarez and Schopf, as well as an additional nonlinear term. This additional nonlinearity increases the period of the resulting oscillation, being closer to the real period of ENSO.